CHAPTER 1 WAVES

'... the chidden billow seems to pelt the clouds ...'

Othello, Act II, Scene I.

Sea waves have attracted attention and comment throughout recorded history. Aristotle (384–322 BC) observed the existence of a relationship between wind and waves, and the nature of this relationship has been a subject of study ever since. However, at the present day, understanding of the mechanism of wave formation and the way that waves travel across the oceans is by no means complete. This is partly because observations of wave characteristics at sea are difficult, and partly because mathematical models of wave behaviour are based upon the dynamics of idealized fluids, and ocean waters do not conform precisely with those ideals. Nevertheless, some facts about waves are well established, at least to a first approximation, and the purpose of this Chapter is to outline the qualitative aspects of water waves and to explore some of the simple relationships of wave dimensions and characteristics.

We start by examining the dimensions of an idealized water wave, and the terminology used for describing waves (Figure 1.1).

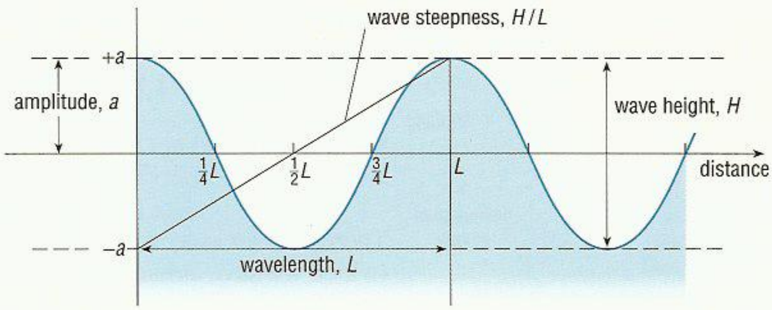


Figure 1.1 Vertical profile of two successive idealized ocean waves, showing their linear dimensions and sinusoidal shape.

Wave height (H) refers to the overall vertical change in height between the wave crest (or peak) and the wave trough. The wave height is twice the wave **amplitude** (a). **Wavelength** (L) is the distance between two successive peaks (or two successive troughs). **Steepness** is defined as wave height divided by wavelength (H/L) and, as can be seen in Figure 1.1, is not the same thing as the slope of the sea-surface between a wave crest and its adjacent trough. The time interval between two successive peaks (or two successive troughs) passing a fixed point is known as the **period** (T), and is generally measured in seconds. The number of peaks (or the number of troughs) which pass a fixed point per second is known as the **frequency** (f).

QUESTION 1.1 If a wave has a frequency of $0.2\,s^{-1}$, what is its period?

As the answer to Question 1.1 shows, period is the reciprocal of frequency. We will return to this concept in Section 1.2.

1.1 WHAT ARE WAVES?

Waves are a common occurrence in everyday life, and are manifested as, for example, sound, the motion of a plucked guitar string, ripples on a pond, or the billows on the ocean. It is not easy to define a wave. Before attempting to do so, let us consider some of the characteristics of wave motion:

1 A wave transfers a disturbance from one part of a material to another. (The disturbance caused by dropping a stone into a pond is transmitted across the pond by ripples.)

2 The disturbance is propagated through the material without any substantial overall motion of the material itself. (A floating cork merely bobs up and down on the ripples, but experiences very little overall movement in the direction of travel of the ripples.)

3 The disturbance is propagated without any significant distortion of the wave form. (A ripple shows very little change in shape as it travels across a pond.)

4 The disturbance appears to be propagated with constant speed.

If the material itself is not being transported by wave propagation, then what *is* being transported?

The answer, 'energy', provides a reasonable working definition of wave motion – a means whereby energy is transported across or through a material without any significant overall transport of the material itself.

So, if energy, and not material, is being transported, what is the nature of the movement observed when ripples cross a pond?

There are two aspects to be considered: first, the progress of the waves (which we have already noted), and secondly, the movement of the water particles themselves. Superficial observation of the effect of ripples on a floating cork suggests that the water particles move 'up and down', but closer observation will reveal that, provided the water is very much deeper than the ripple height, the cork is describing a nearly circular path in a vertical plane, parallel with the direction of wave movement. In a more general sense, the particles are displaced from an equilibrium position, and a wave motion is the propagation of regular oscillations about that equilibrium position. Thus, the particles experience a displacing force and a restoring force. The nature of these forces is often used in the descriptions of various types of waves.

1.1.1 TYPES OF WAVES

All waves can be regarded as **progressive waves**, in that energy is moving through, or across the surface of, the material. The so-called **standing wave**, of which the plucked guitar string is an example, can be considered as the sum of two progressive waves of equal dimensions, but travelling in opposite directions. We examine this in more detail in Section 1.6.4.

Waves which travel through the material are called body waves. Examples of body waves are sound waves and seismic P- and S-waves, but our main concern in this Volume is with *surface waves* (Figure 1.2). The most familiar surface waves are those which occur at the interface between atmosphere and ocean, caused by the wind blowing over the sea. Other external forces acting on the fluid can also generate waves. Examples range from raindrops falling into tidal pools, through diving gannets and ocean-going liners to earthquakes (see Section 1.6.3).

The tides are also waves (Figure 1.2), caused by the gravitational influence of the Sun and Moon and having periods corresponding to the causative forces. This aspect is considered in more detail in Chapter 2. Most other waves, however, result from a non-periodic disturbance of the water. The water particles are displaced from an equilibrium position, and to regain that position they require a restoring force, as mentioned above. The restoring force causes a particle to 'overshoot' on either side of the equilibrium position. Such alternate displacements and restorations establish a characteristic oscillatory 'wave motion', which in its simplest form has sinusoidal characteristics (Figures 1.1 and 1.6), and is sometimes referred to as simple harmonic motion. In the case of surface waves on water, there are two such restoring forces which maintain wave motion:

1 The gravitational force exerted by the Earth.

2 Surface tension, which is the tendency of water molecules to stick together and present the smallest possible surface to the air. So far as the effect on water waves is concerned, it is as if a weak elastic skin were stretched over the water surface.

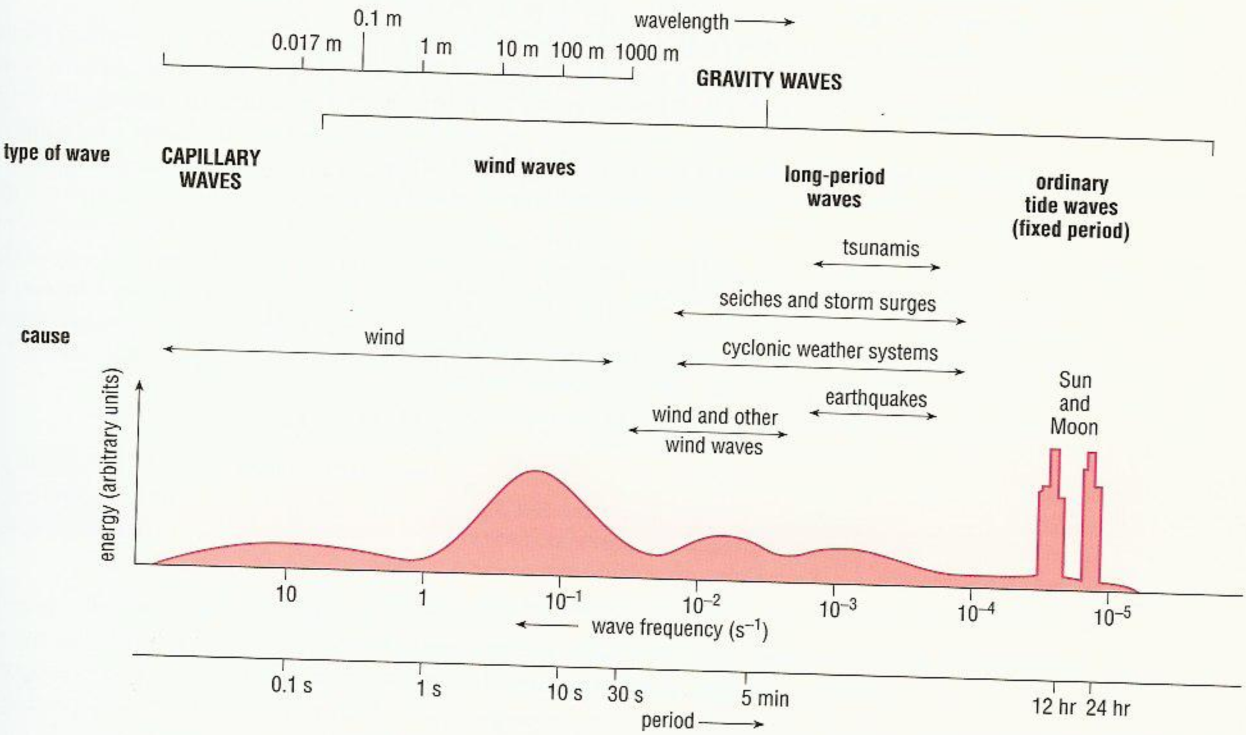


Figure 1.2 Types of surface waves, showing the relationships between wavelength, wave frequency and period, the nature of the forces that cause them, and the relative amounts of energy in each type of wave. Unfamiliar terms will be explained later. *Note:* Waves caused by 'other wind waves' are waves resulting from interactions between waves of higher frequency as they move away from storm areas – see Section 1.4.2.

Water waves are affected by both of these forces. In the case of waves with wavelengths less than about 1.7 cm, the principal restoring force is surface tension, and such waves are known as **capillary waves**. They are important in the context of remote sensing of the oceans (Section 1.7.1). However, the main interest of oceanographers lies with surface waves of wavelengths greater than 1.7 cm, and the principal restoring force for such waves is gravity; hence they are known as **gravity waves** (Figure 1.2).

Gravity waves can also be generated at an interface between two layers of ocean water of differing densities. Because the interface is a surface, such waves are, strictly speaking, surface waves, but oceanographers usually refer to them as **internal waves**. These occur most commonly where there is a rapid increase of density with depth, i.e. a steep density gradient, or **pycnocline**. Pycnoclines themselves result from steep gradients of temperature and/or salinity, the two properties which together govern the density of seawater. Because the difference in density between two water layers is much smaller than that between water and air, less energy is required to displace the interface from its equilibrium position, and oscillations are more easily set up at an internal interface than at the sea-surface. Internal waves travel considerably more slowly than most surface waves. They have greater amplitudes than all but the largest surface waves (up to a few tens of metres), as well as longer periods (minutes or hours rather than seconds, cf. Figure 1.2) and longer wavelengths (hundreds rather than tens of metres). Internal waves are of considerable importance in the context of vertical mixing processes in the oceans, especially when they break.

Not all waves in the oceans are displaced primarily in a vertical plane. For example, because atmosphere and oceans are on a rotating Earth, variation of **planetary vorticity** with latitude (i.e. variation in the angular velocity of the Earth's surface and hence in the effect of the Earth's rotation on horizontal motions) causes horizontal deflection of atmospheric and oceanic currents, and provides restoring forces which establish oscillations mainly in a horizontal plane, so that easterly or westerly currents tend to swing back and forth about an equilibrium latitude. These large-scale horizontal oscillations are known as **planetary** (or **Rossby**) **waves**, and may occur as surface or as internal waves. They are not gravity waves (i.e. the restoring force is not gravity) and so do not appear in Figure 1.2.

1.1.2 WIND-GENERATED WAVES IN THE OCEAN

In 1774, Benjamin Franklin said: 'Air in motion, which is wind, in passing over the smooth surface of the water, may rub, as it were, upon that surface, and raise it into wrinkles, which, if the wind continues, are the elements of future waves'.

In other words, if two fluid layers having differing speeds are in contact, there is frictional stress between them and there is a transfer of momentum and energy. The frictional stress exerted by a moving fluid is proportional to the *square* of the speed of the fluid, so the **wind stress** exerted upon a water surface is proportional to the square of the wind speed. At the sea-surface, most of the transferred energy results in waves, although a small proportion is manifest as wind-driven currents. In 1925, Harold Jeffreys suggested that waves obtain energy from the wind by virtue of pressure differences caused by the sheltering effect provided by wave crests (Figure 1.3).

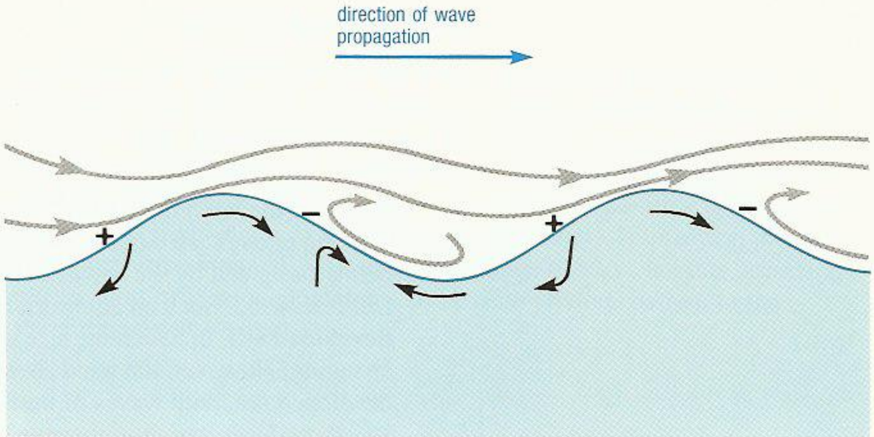


Figure 1.3 Jeffreys' 'sheltering' model of wave generation. Curved grey lines indicate air flow; shorter, black arrows show water movement. The rear face of the wave against which the wind blows experiences a higher pressure than the front face, which is sheltered from the force of the wind. Air eddies are formed in front of each wave, leading to excesses and deficiencies of pressure (shown by plus and minus signs respectively), and the pressure difference pushes the wave along.

Although Jeffreys' hypothesis fails to explain the formation of very small waves, it does seem to work if:

1 Wind speed exceeds wave speed.

2 Wind speed exceeds 1 m s^{-1}.

3 The waves are steep enough to provide a sheltering effect.

Empirically, it can be shown that the sheltering effect is at a maximum when wind speed is approximately three times the wave speed. In general, the greater the amount by which wind speed exceeds wave speed, the steeper the wave. In the open oceans, most wind-generated waves have steepness (H/L) of about 0.03 to 0.06. However, as we shall see later, wave speed in deep water is not related to wave steepness, but to wavelength – the greater the wavelength, the faster the wave travels.

QUESTION 1.2 Two waves have the same height, but differing steepness. Which of the two waves will travel the faster?

Consider the sequence of events that occurs if, after a period of calm weather, a wind starts to blow, rapidly increases to a gale, and continues to blow at constant gale force for a considerable time. No significant wave growth occurs until wind speed exceeds 1 m s^{-1}. Then, small steep waves form as the wind speed increases. Even after the wind has reached a constant gale force, the waves continue to grow with increasing rapidity until they reach a size and wavelength appropriate to a speed which corresponds to one-third of the wind speed. Beyond this point, the waves continue to grow in size, wavelength and speed, but at an ever-diminishing rate. On the face of it, one might expect that wave growth would continue until wave speed was the same as wind speed. However, in practice wave growth ceases whilst wave speed is still at some value below wind speed. This is because:

1 Some of the wind energy is transferred to the ocean surface via a tangential force, producing a surface current.

2 Some wind energy is dissipated by friction, and is converted to heat and sound.

3 Energy is lost from larger waves as a result of **white-capping**, i.e. breaking of the tip of the wave crest because it is being driven forward by the wind faster than the wave itself is travelling. Much of the energy dissipated during white-capping is converted into forward momentum of the water itself, reinforcing the surface current initiated by process 1 above.

1.1.3 THE FULLY DEVELOPED SEA

We have already seen that the size of waves in deep water is governed not only by the actual wind speed, but also by the length of time the wind has been blowing at that speed. Wave size also depends upon the unobstructed distance of sea, known as the **fetch**, over which the wind blows.

Provided the fetch is extensive enough and the wind blows at constant speed for long enough, an equilibrium is eventually reached, in which energy is being dissipated by the waves at the same rate as the waves receive energy from the wind. Such an equilibrium results in a sea state called a **fully developed sea**, in which the size and characteristics of the waves are not changing. However, the wind speed is usually variable, so the ideal fully developed sea, with waves of uniform size, rarely occurs. Variation in wind speed produces variation in wave size, so, in practice, a fully developed sea consists of a range of wave sizes known as a **wave field**. Waves coming into the area from elsewhere will also contribute to the range of wave sizes, as will interaction between waves – a process we explain in Section 1.4.2.

Oceanographers find it convenient to consider a wave field as a spectrum of wave energies (Figure 1.4). The energy contained in an individual wave is proportional to the square of the wave height (see Section 1.4).

QUESTION 1.3 Examine Figure 1.4. Does the energy contained in a wave field increase or decrease as the average frequency of the constituent waves increases?

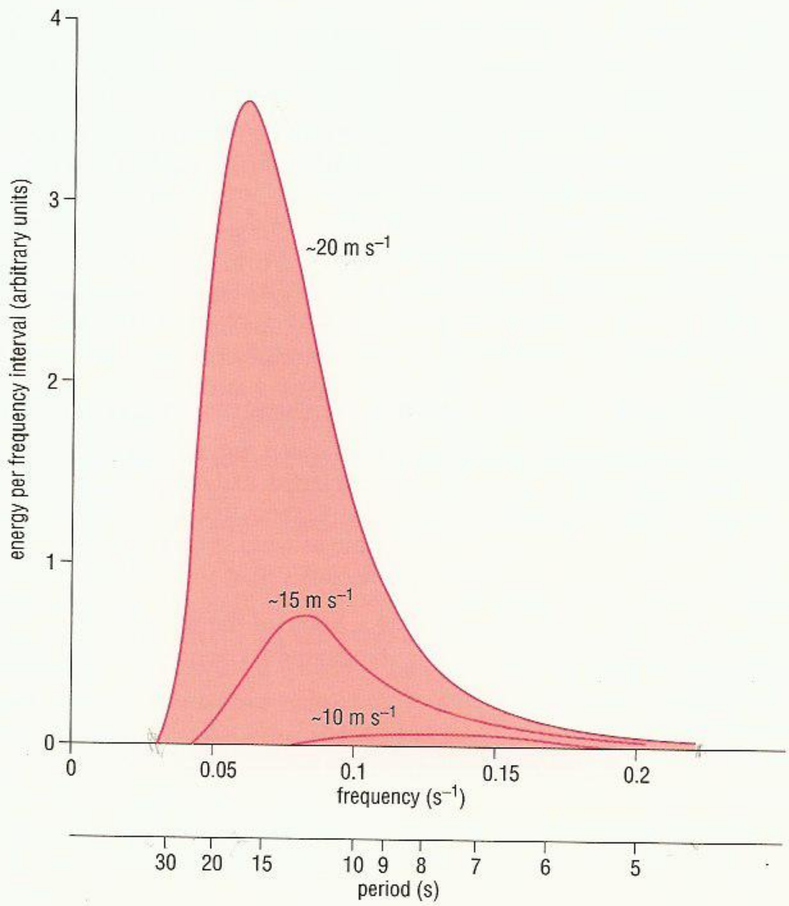


Figure 1.4 Wave energy spectra for three fully developed seas, related to wind speeds of 20, 30 and 40 knots (about 10, 15 and 20 m s^{-1} respectively). The area under each curve is a measure of the total energy in that particular wave field.

1.1.4 WAVE HEIGHT AND WAVE STEEPNESS

As was hinted in Section 1.1.3, the height of any real wave is determined by many component waves, of different frequencies and amplitudes, which move into and out of phase with, and across each other ('in phase' means that peaks and troughs coincide). In theory, if the heights and frequencies of all the contributing waves were known, it would be possible to predict the heights and frequencies of the real waves accurately. In practice, this is rarely possible. Figure 1.5 illustrates the range of wave heights occurring over a short time at one location – there is no obvious pattern to the variation of wave height.

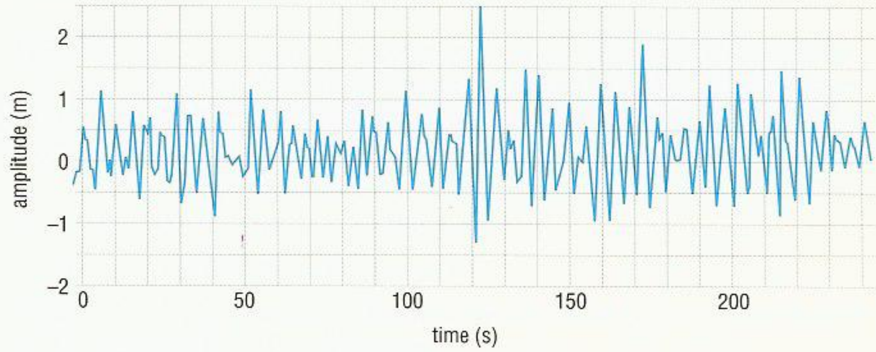


Figure 1.5 A typical wave record, i.e. a record of variation in water level (displacement from equilibrium) with time at one position.

For many applications of wave research, it is necessary to choose a single wave height which characterizes a particular sea state. Many oceanographers use the **significant wave height**, $H_{1/3}$, which is the average height of the highest one-third of all waves occurring in a particular time period. In any wave record, there will also be a maximum wave height, H_{max}. Prediction of H_{max} for a given period of time has great value in the design of structures such as flood barriers, harbour installations and drilling platforms. To build these structures with too great a margin of safety would be unnecessarily expensive, but to underestimate H_{max} could have tragic consequences. However, it is necessary to emphasize the essentially random nature of H_{max}. Although the wave $H_{max(25\ years)}$, will occur *on average* once every 25 years, this does not mean such a wave will automatically occur every 25 years – there may be periods much longer than that without one. On the other hand, two such waves might appear next week.

As wind speed increases, so $H_{1/3}$ in the fully developed sea increases. The relationship between sea state, $H_{1/3}$ and wind speed is expressed by the **Beaufort Scale** (Table 1.1, overleaf). The Beaufort Scale can be used to estimate wind speed at sea, but is valid only for waves generated within the local weather system, and assumes that there has been sufficient time for a fully developed sea to have become established (cf. Figure 1.4).

The absolute height of a wave is less important to sailors than is its steepness (H/L). As mentioned in Section 1.1.2, most wind-generated waves have a steepness in the order of 0.03 to 0.06. Waves steeper than this can present problems for shipping, but fortunately it is very rare for wave steepness to exceed 0.1. In general, wave steepness diminishes with increasing wavelength. The short choppy seas rapidly generated by local squalls are particularly unpleasant to small boats because the waves are steep, even though not particularly high. On the open ocean, very high waves can usually be ridden with little discomfort because of their relatively long wavelengths.

Table 1.1 A selection of information from the Beaufort Wind Scale.

Beaufort No.	Name	Wind speed (mean) knots	Wind speed (mean) $m\,s^{-1}$	State of the sea-surface	Significant wave height, $H_{1/3}$ (m)
0	Calm	<1	0.0–0.2	Sea like a mirror	0
1	Light air	1–3	0.3–1.5	Ripples with appearance of scales; no foam crests	0.1–0.2
2	Light breeze	4–6	1.6–3.3	Small wavelets; crests have glassy appearance but do not break	0.3–0.5
3	Gentle breeze	7–10	3.4–5.4	Large wavelets; crests begin to break; scattered white horses	0.6–1.0
4	Moderate breeze	11–16	5.5–7.9	Small waves, becoming longer; fairly frequent white horses	1.5
5	Fresh breeze	17–21	8.0–10.7	Moderate waves taking longer form; many white horses and chance of some spray	2.0
6	Strong breeze	22–27	10.8–13.8	Large waves forming; white foam crests extensive everywhere and spray probable	3.5
7	Near gale	28–33	13.9–17.1	Sea heaps up and white foam from breaking waves begins to be blown in streaks; spindrift begins to be seen	5.0
8	Gale	34–40	17.2–20.7	Moderately high waves of greater length; edges of crests break into spindrift; foam is blown in well-marked streaks	7.5
9	Strong gale	41–47	20.8–24.4	High waves; dense streaks of foam; sea begins to roll; spray may affect visibility	9.5
10	Storm	48–55	24.5–28.4	Very high waves with overhanging crests; sea-surface takes on white appearance as foam in great patches is blown in very dense streaks; rolling of sea is heavy and visibility reduced	12.0
11	Violent storm	56–64	28.5–32.7	Exceptionally high waves; sea covered with long white patches of foam; small and medium-sized ships might be lost to view behind waves for long times; visibility further reduced	15.0
12	Hurricane	>64	>32.7	Air filled with foam and spray; sea completely white with driving spray; visibility greatly reduced	>15

1.2 SURFACE WAVE THEORY

To simplify the theory of surface waves, we assume here that the wave-form is sinusoidal and can be represented by the curves shown in Figures 1.1 and 1.6. This assumption allows us to consider wave **displacement** (η) as simple harmonic motion, i.e. a sinusoidal variation in water level caused by the wave's passage. Figure 1.1 shows how the displacement varies over distance at a fixed instant in time – a 'snapshot' of the passing waves – whereas Figure 1.6 shows how wave displacement varies with time at a fixed point.

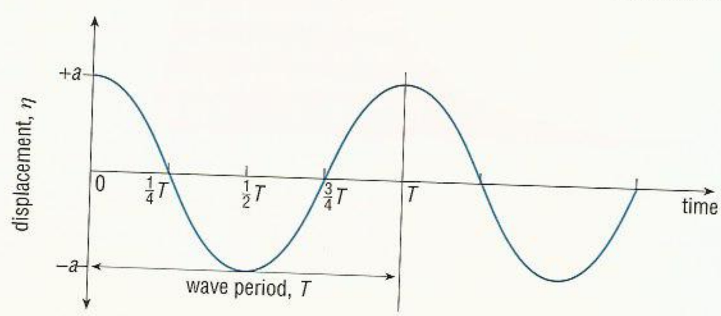


Figure 1.6 The displacement of an idealized wave at a fixed point, plotted against time. Maximum and minimum displacements are recorded in fractions of the period, T.

Before examining displacement, let us remind ourselves of the relationship between period and frequency.

QUESTION 1.4 If 16 successive wave troughs pass a fixed point during a time interval of one minute and four seconds, what is the frequency of the waves?

The displacement (η) of a wave at a fixed instant in time, or at a fixed point in space, varies between $+a$ (at the peak) and $-a$ (in the trough). Displacement is zero where $L = \frac{1}{4}L$ on Figure 1.1 (and at intervals of $L/2$ along the distance axis). Displacement is also zero at $T = \frac{1}{4}T$ on Figure 1.6 (and at intervals of $T/2$ along the time axis).

QUESTION 1.5 Use Figure 1.6 to help you answer the following questions. The peak, or crest, of a wave having a wavelength of 624 m, a frequency of $0.05\ s^{-1}$, and travelling in deep water, passes a fixed point P. What is the displacement at P (in terms of the amplitude, a):

(a) 30 seconds after the peak has passed?

(b) 80 seconds after the peak has passed?

(c) 85 seconds after the peak has passed?

What is the displacement at a second point, Q, which is 312 m away from P in the direction of wave propagation:

(d) when the displacement at P is zero?

(e) when the displacement at P is $-a$?

(f) 5 seconds after a trough has passed P?

The curves shown in Figures 1.1 and 1.6 are both sinusoidal. However, most wind-generated waves do not have simple sinusoidal forms. The steeper the wave, the further it departs from a simple sine curve. Very steep waves resemble a trochoidal curve, which is illustrated in Figure 1.7.

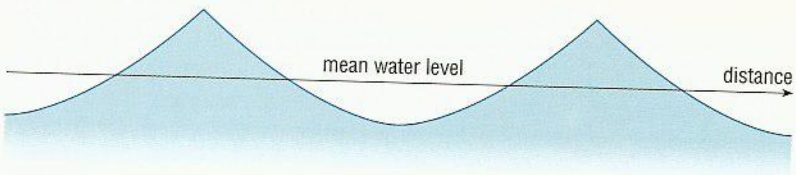


Figure 1.7 Profile of trochoidal waves.

A point marked on the rim of a car tyre will appear to trace out a trochoidal curve as the car is driven past an observer. Invert that pattern and you have the profile of a trochoidal water wave. We do not need to delve into the mathematical complexities of trochoidal wave forms here, because the sinusoidal model is sufficient for our purposes.

1.2.1 MOTION OF WATER PARTICLES

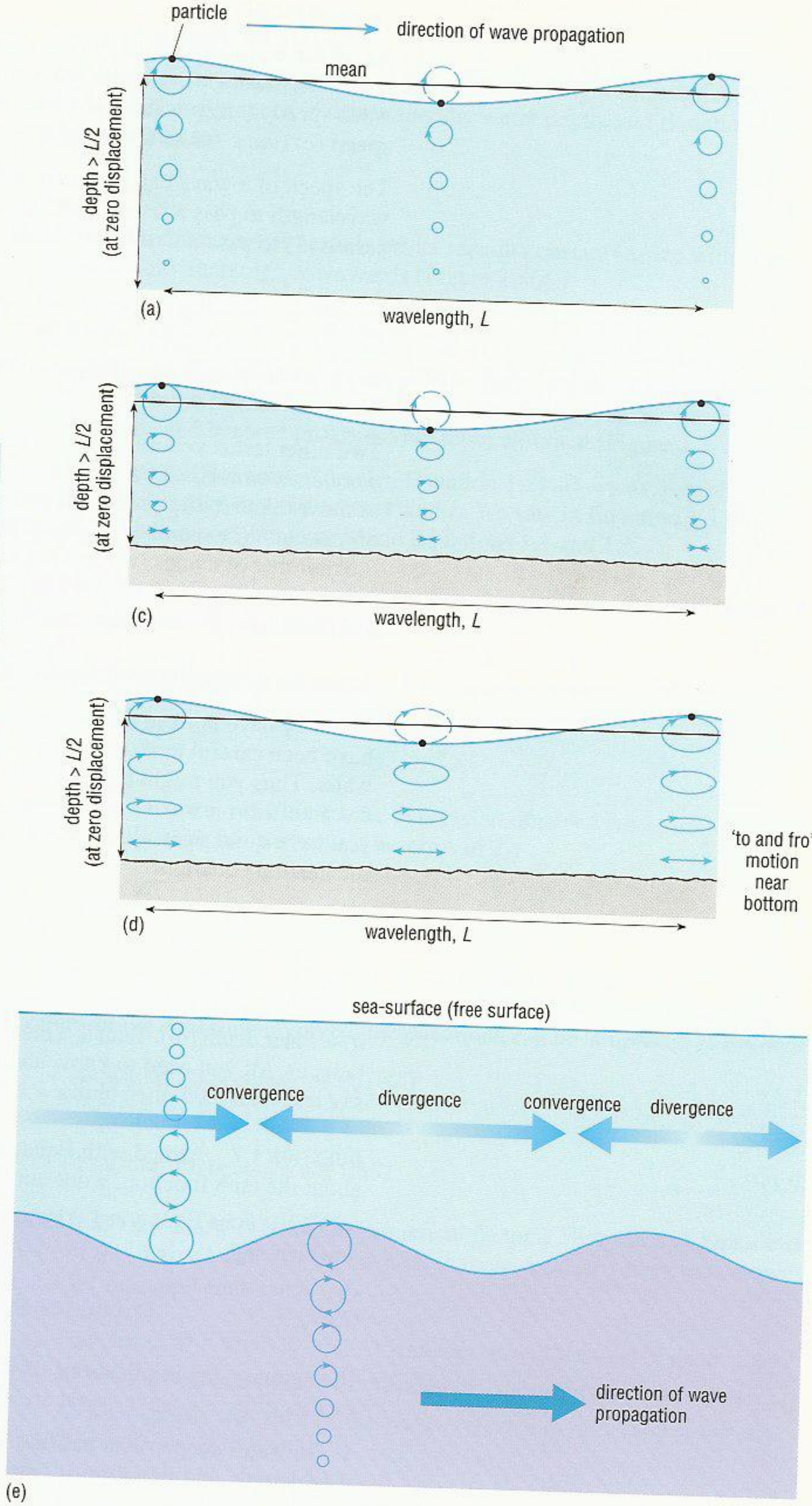
Water particles in a wave in deep water move in an almost closed circular path. At wave crests, the particles are moving in the same direction as wave propagation, whereas in the troughs they are moving in the opposite direction. At the surface, the orbital diameter corresponds to wave height, but the diameters decrease exponentially with increasing depth, until at a depth roughly equal to half the wavelength, the orbital diameter is negligible, and there is virtually no displacement of the water particles (Figure 1.8(a)). This has some important practical applications. For example, a submarine only has to submerge about 150 m to avoid the effects of even the most severe storm at sea, and knowledge of the exponential decrease of wave influence with depth has implications for the design of stable floating oil rigs.

It is important to realize that the orbits are only approximately circular. There is a small net component of forward motion, particularly in waves of large amplitude, so that the orbits are not quite closed, and the water, whilst in the crests, moves slightly further forward than it moves backward whilst in the troughs. This small net forward displacement of water in the direction of wave travel is termed **wave drift** (see Figure 1.8(b)). In shallow water, where depth is less than half the wavelength and the waves 'feel' the sea-bed, the orbits become progressively flattened with depth (Figure 1.8(c) and (d)). The significance of these changes will be seen in Section 1.5, and in the Chapters on sediment movement.

The orbital motions relevant to internal waves (Section 1.1.1) are shown in Figure 1.8(e): they are in opposite directions on either side of the interface. The passage of internal waves can often be detected visually by secondary effects at the surface, especially if the upper layer (above the pycnocline) is not very thick and the sea is relatively calm. As the internal waves travel along, the upper layer becomes alternately thinner (over the internal wave crests) and thicker (over the troughs). The result is that there are **convergences** and **divergences** of water at the surface, as water is displaced back and forth between the thinner and thicker parts of the upper layer.

A combination of these to-and-fro motions and the opposing particle orbits on either side of the interface may, under certain conditions, influence the movement of vessels with a draught comparable to the depth to the interface, sometimes causing an unexpected drag on the hull, thus making the vessels sluggish and difficult to handle.

Sometimes the convergences compress short wavelength surface waves, making them more visible, but commonly they bring together organic material (especially oils released by marine organisms), which increases the surface tension and tends to suppress ripples, so that the water is smoother than elsewhere. Alternating bands of smooth and rippled surface water at intervals of a few hundred metres may thus indicate the passage of an internal wave, but whether ripples represent convergences or divergences depends upon local conditions.



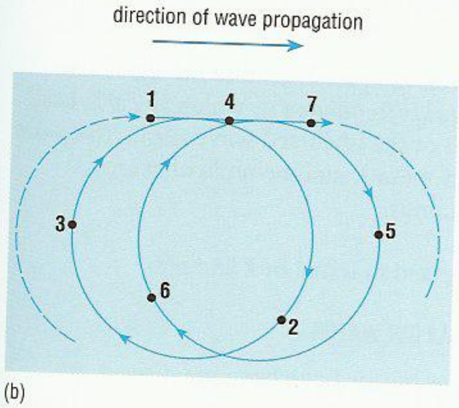


Figure 1.8 (a) Particle motion in deep-water waves (depth greater than $L/2$), showing exponential decrease of the diameters of the orbital paths with depth.
(b) Particle motion in large deep-water waves, showing wave drift.
(c) Particle motion in waves where water depth is less than $L/2$ but greater than $L/20$ (see Section 1.2.3), showing both decrease in horizontal orbital diameter and progressive flattening of the orbits near the sea-bed.
(d) Particle motion in shallow-water waves (depth less than $L/20$, see Section 1.2.3), showing progressive flattening of the orbits (but no decrease in horizontal diameter) near the sea-bed.
(e) Particle motion in internal waves (Section 1.1.1), and the convergences and divergences associated with their passage. The orbits will only be truly circular if the layers are thick enough (i.e. greater than half the wavelength). The orbital diameters decrease linearly (i.e. not exponentially) with distance from the interface but may not reach zero at the free surface above the interface (where any undulations do not necessarily reflect those of the internal waves). Orbital motions are in *opposite* directions above and below the interface.

1.2.2 WAVE SPEED

As we have already hinted, there are mathematical relationships linking the characteristics of wavelength (L), wave period (T) and wave height (H) to wave speed in deep water and to wave energy. First, let us consider **wave speed** (c) (the c stands for '*celerity of propagation*').

The speed of a wave can be ascertained from the time taken for one wavelength to pass a fixed point. As one wavelength (L) takes one wave period (T) to pass a fixed point, then:

$$c = L/T \tag{1.1}$$

which is simply a form of the well-known expression: speed = distance/time.

So, if we know any two of the variables in Equation 1.1, we can calculate the third.

Two other terms you may meet in oceanographic literature are the *wave number, k*, which is $2\pi/L$, and the *angular frequency* σ, which is $2\pi/T$; both of these relate to the sinusoidal nature of the idealized wave form. The units of k are m^{-1} (i.e. number of waves per metre), and the units of σ are s^{-1} (i.e. number of cycles (waves) per second).

QUESTION 1.6 How would c be expressed in terms of k and σ?

1.2.3 WAVE SPEED IN DEEP AND IN SHALLOW WATER

You may have noticed that when wave speeds have been mentioned we have been careful to state that the waves described were travelling in deep water. Thus you might have suspected that in shallow water, water depth has an effect on wave speed, because of interaction with the sea-bed. If so, you were quite right. Wave speed in any water depth can be represented by the general equation:

$$c = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)} \tag{1.2}$$

where the acceleration due to gravity $g = 9.8$ m s^{-2}, L = wavelength (m), and d = water depth (m). Tanh is a mathematical function known as the hyperbolic tangent. All you need to know about it in this context is that if x is small, say less than 0.05, then $\tanh x \approx x$, and if x is larger than π, then $\tanh x \approx 1$.

QUESTION 1.7 Armed with Equation 1.2, and the information given above about the tanh function, work out the answers to the following questions:

(a) What does Equation 1.2 become if the water depth exceeds half the wavelength?

(b) What does Equation 1.2 become if the water depth is very much smaller than L?

In summary, the implications of your answers to Question 1.7 in terms of factors affecting wave speed are as follows (cf. Figure 1.8):

1 In water deeper than half the wavelength, wave speed depends upon the wavelength, and Equation 1.2 approximates to:

$$c = \sqrt{\frac{gL}{2\pi}} \tag{1.3}$$

Diameters of circular particle orbits decrease exponentially downwards to near zero at depth = $L/2$ (Figure 1.8(a)).

2 In water very much shallower than the wavelength (in practice, when $d < L/20$), wave speed is determined by water depth, and Equation 1.2 approximates to:

$$c = \sqrt{gd} \tag{1.4}$$

Horizontal diameters of particle orbits remain constant in size with depth, but ellipticity increases downwards (Figure 1.8(d)).

3 When d lies between $L/20$ and $L/2$, the full form of Equation 1.2 is required. Hence, to calculate wave speed you would need to know wavelength and depth, and have access to a set of hyperbolic tangent tables, or a calculator with hyperbolic functions on its keyboard. Particle orbits decrease in size downwards and become progressively more elliptical (Figure 1.8(c)).

The answer to Question 1.7(a) (i.e. Equation 1.3) allows us to explore further the relationships between T and L. We saw in Equation 1.1 that $c = L/T$, so it is possible to combine Equations 1.1 and 1.3.

QUESTION 1.8 Derive an equation for wavelength (L) in terms of period (T), using Equations 1.1 and 1.3.

The answer to Question 1.8 provides an equation expressing L in terms of T, i.e.

$$L = \frac{gT^2}{2\pi} \tag{1.5}$$

A similar exercise, substituting the expression obtained for L from Equation 1.5 into Equation 1.1, will give c in terms of T:

$$c = \frac{gT}{2\pi} \tag{1.6}$$

Thus, it is possible, given only one of the wave characteristics c, T or L, to calculate either of the other two. Moreover, by substituting the numerical values of the constants involved, the equations can be simplified as follows:

Equation 1.3 becomes $c = \sqrt{1.56L}$ (1.7)

Equation 1.5 becomes $L = 1.56T^2$ (1.8)

Equation 1.6 becomes $c = 1.56T$ (1.9)

QUESTION 1.9 Show (a) how the numerical factor 1.56 in each of Equations 1.7 to 1.9 is derived, and (b) that the units in those equations work out correctly.

QUESTION 1.10

(a) The period of a wave is 20 s. At what speed will it travel in deep water?

(b) At what speed will a wave of wavelength 312 m travel in deep water?

(c) At what speeds will each of the waves referred to in (a) and (b) above travel in water of 12 m depth?

The answer to Question 1.10(c) highlights an important conclusion about wave speed in shallow water. *Provided that depth is less than 1/20 of their wavelengths*, all waves will travel at the same speed in water of a given depth.

1.2.4 ASSUMPTIONS MADE IN SURFACE WAVE THEORY

The simple wave theory introduced above is a first-order approximation, and makes the following assumptions:

1 The wave shapes are sinusoidal.

2 The wave amplitudes are very small compared with wavelengths and depths.

3 Viscosity and surface tension can be ignored.

4 The Coriolis force (see Section 2.3) and vorticity (Section 1.1.1), which result from the Earth's rotation, can be ignored.

5 The depth is uniform, and the bottom has no bumps or hummocks.

6 The waves are not constrained or deflected by land masses, or by any other obstruction.

7 That real three-dimensional waves behave in a way that is analogous to a two-dimensional model.

None of the above assumptions is valid in the strictest sense, but predictions based on simple models of surface wave behaviour approximate closely to how wind-generated waves behave in practice.

1.3 WAVE DISPERSION AND GROUP SPEED

Those deep-water waves that have the greatest wavelengths and longest periods travel fastest, and thus are first to arrive in regions distant from the storm which generated them. This separation of waves by virtue of their differing rates of travel is known as **dispersion**, and Equation 1.3 ($c = \sqrt{gL / 2\pi}$) is sometimes known as the *dispersion equation*, because it shows that waves of longer wavelength (L) travel faster than shorter wavelength waves.

The simple experiment of tossing a stone into a still pond shows that a band of ripples is created, which gets wider with increasing distance from the original disturbance. Ripples of longer wavelength progressively out-distance shorter ones – an example of dispersion in action. There is a second feature of the ripple band, which is not obvious at first sight. Each individual ripple travels faster than the band of ripples as a whole. A ripple appears at the back of the band, travels through it, and disappears out of the front. The speed of the band, called the **group speed**, is about half the wave speed of the individual ripples which travel through that band.

To understand the relationship between wave speed and group speed, the additive effect of two sets of waves (or *wave trains*) needs to be examined. If the difference between the wavelengths of two sets of waves is relatively small, the two sets will 'interfere' and produce a single set of resultant waves. Figure 1.9 shows a simplified and idealized example of interference. Where the crests of the two wave trains coincide (i.e. they are 'in phase'), the wave amplitudes are added, and the resultant wave has about twice the amplitude of the two original waves. Where the two wave trains are 'out of phase', such that the crests of one wave train coincide with the troughs of the other, the amplitudes cancel out, and the water surface has minimal displacement.

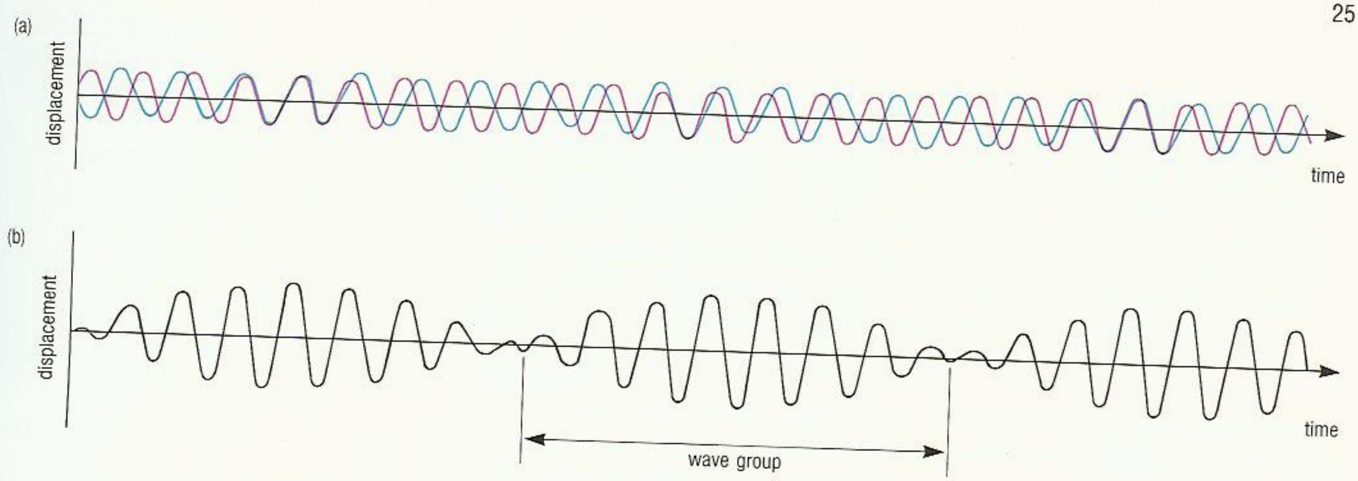


Figure 1.9 (a) The merging of two wave trains (shown in red and blue) of slightly different wavelengths (but the same amplitudes), to form wave groups (b).

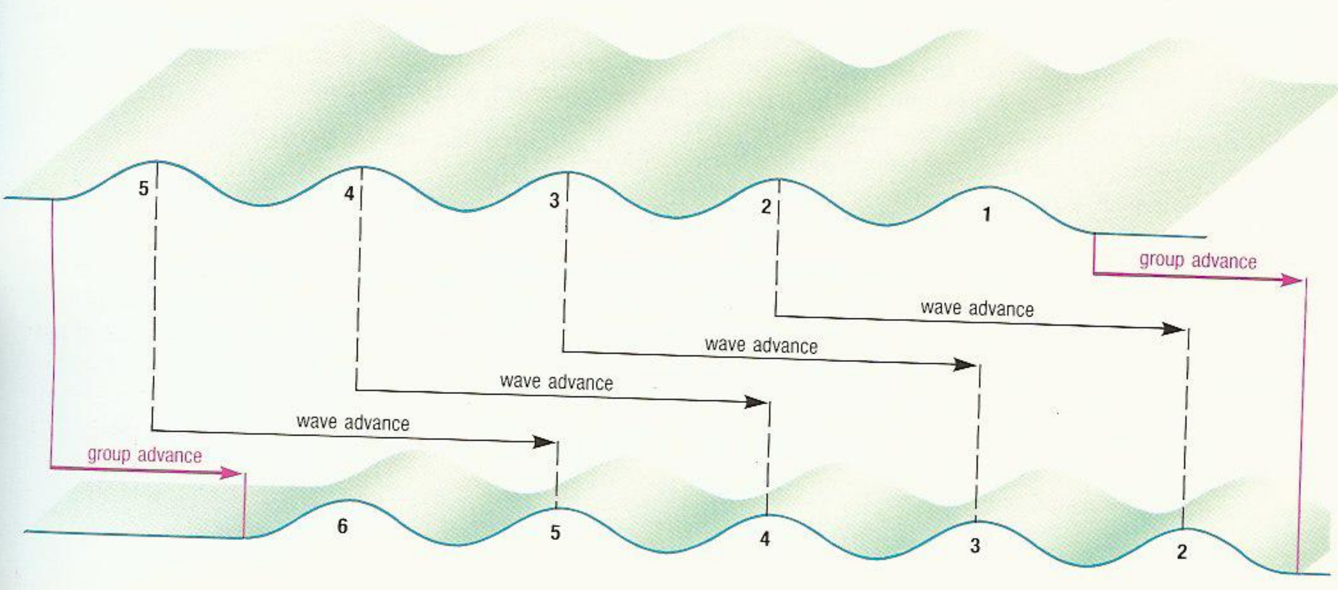
The two component wave trains thus interact, each losing its individual identity, and combine to form a series of wave groups, separated by regions almost free from waves. The wave group advances more slowly than individual waves in the group, and thus in terms of the occurrence and propagation of waves, group speed is more significant than speeds of the individual waves. Individual waves do not persist for long in the open ocean, only as long as they take to pass through the group. Figure 1.10 shows the relationship between wave speed (sometimes called *phase speed*) and group speed in the open ocean.

The group speed is *half* the average speed of the two wave trains, and for your interest we present below an abbreviated form of how this relationship is derived. It is not necessary to follow all the steps that lead to Equation 1.10 (overleaf), still less to memorize them.

If two sets of waves are interfering to produce a succession of wave groups, the group speed (c_g) is the difference between the two angular frequencies (σ_1 and σ_2) divided by the difference between the two wave numbers (k_1 and k_2 respectively), i.e.

$$c_g = \frac{\sigma_1 - \sigma_2}{k_1 - k_2}$$

Figure 1.10 The relationship between wave speed (phase speed) and group speed. As the wave advances from left to right, each wave moves through the group to die out at the front (e.g. wave 1), as new waves form at the rear (e.g. wave 6). In this process, the distance travelled by each individual wave as it moves from the rear to front of the group is twice that travelled by the group as a whole. Hence, the wave speed is twice that of the group speed.



We have seen (Question 1.6) that $c = \sigma/k$, and we know (Section 1.2.2) that angular frequency, σ, can be expressed in terms of T, also that wave number k, can be expressed in terms of L. In addition, we know that Equations 1.6 and 1.3, respectively, enable us to express both T and L in terms of c (Section 1.2.3). Hence, c_g can be expressed in terms of the respective speeds, c_1 and c_2, of the two wave trains. The equation obtained is:

$$c_g = \frac{c_1 \times c_2}{c_1 + c_2}$$

If c_1 is nearly equal to c_2, this equation simplifies to:

$$c_g \approx c^2/2c$$

$$\text{or } c_g \approx c/2 \tag{1.10}$$

where c is the average speed of the two wave trains.

What happens to group speed when waves enter shallow water?

Equation 1.2 shows that as the water becomes shallower, wavelength becomes less important, and depth more important, in determining wave speed. As a result, in shoaling water, wave (phase) speed decreases, becoming closer to group speed. Eventually, at depths less than $L/20$, all waves travel at the same depth-determined speed, there will be no wave–wave interference, and therefore in effect each wave will represent its own 'group'. Thus, in shallow water, group speed can be regarded as equal to wave (phase) speed.

1.4 WAVE ENERGY

The energy possessed by a wave is in two forms:

1 kinetic energy, which is the energy inherent in the orbital motion of the water particles; and

2 potential energy possessed by the particles as a result of being displaced from their mean (equilibrium) position.

For a water particle in a given wave, energy is continually being converted from potential energy (at crest and trough) to kinetic energy (as it passes through the equilibrium position), and back again.

The total energy (E) *per unit area* of a wave is given by:

$$E = \frac{1}{8}(\rho g H^2) \tag{1.11}$$

where ρ is the density of the water (in kg m^{-3}), g is 9.8 m s^{-2}, and H is the wave height (m). The energy (E) is then in joules per square metre (J m^{-2}). Equation 1.11 shows that wave energy is proportional to the square of the wave height.

QUESTION 1.11 Would the total energy of a wave be doubled if its amplitude were doubled?

1.4.1 PROPAGATION OF WAVE ENERGY

Figures 1.9 and 1.10 show that waves travel in groups in deep water, with areas of minimal disturbance between groups. Individual waves die out at the front of each group. It is obvious that no energy is being transmitted across regions where there are no waves, i.e. between the groups. It follows that the energy is contained within the wave group, and travels at the group speed. The *rate* at which energy is supplied at a particular location (e.g. a beach) is called **wave power**, and is the product of group speed (c_g) and wave energy per unit area (E), expressed *per unit length of wave crest*.

QUESTION 1.12

(a) In the case of waves in deep water, what is the energy per square metre of a wave field made up of waves with an average amplitude of 1.3 m? (Use $\rho = 1.03 \times 10^3$ kg m^{-3}.)

(b) What would be the wave power (in kW per metre of crest length) if the waves had a steepness of 0.04? (1 watt = 1 J s^{-1}, and one kilowatt (kW) = 10^3 W.)

1.4.2 ATTENUATION OF WAVE ENERGY

Wave **attenuation** involves loss or dissipation of wave energy, resulting in a reduction of wave height. Energy is dissipated in four main ways:

1 White-capping, which involves transfer of wave energy to the kinetic energy of moving water, thus reinforcing the wind-driven surface current (Section 1.1.2).

2 Viscous attenuation, which is only important for very high frequency capillary waves, and involves dissipation of energy into heat by friction between water molecules.

3 Air resistance, which applies to large steep waves soon after they leave the area in which they were generated and enter regions of calm or contrary winds.

4 **Non-linear wave–wave interaction**, which is more complicated than the simple (linear) combination of frequencies to produce wave groups as outlined in Section 1.3.

Non-linear interaction appears to be most important in the frequency range 0.2 to 0.3 s^{-1}. Groups of three or four frequencies can interact in complex non-linear ways, to transfer energy to waves of both higher and lower frequencies. A rough but useful analogy is that of the collision of two drops of water. A linear combination would simply involve the two drops coalescing (adding together) into one big drop, whereas a non-linear combination is akin to a collision between the drops so that they split into a number of drops of differing sizes. The total amount of water in the drops (analogous to the total amount of energy in the waves) is the same before and after the collision.

Thus, non-linear wave–wave interaction involves no loss of energy in itself, because energy is simply 'swapped' between different frequencies. However, the total amount of energy available for such 'swapping' will gradually decrease, because higher frequency waves are more likely to dissipate energy in the ways described under 1 and 2 above. For example, higher frequency waves are likely to be steep, and thus more prone to white-capping. Wave attenuation is greatest in the storm-generating area, where there are waves of many frequencies, and hence more opportunities for energy exchange between them.

1.4.3 SWELL

The sea-surface is rarely still. Even when there is no wind, and the sea 'looks like a mirror', a careful observer will notice waves of very long wavelength (say 300 to 600 m) and only a few centimetres amplitude. At other times, a sea may include locally generated waves of small wavelength, and travelling through these waves, possibly at a large angle to the wind, other waves of much greater wavelength. Such long waves are known as **swell**, which is simply defined as waves that have been generated elsewhere and have travelled far from their place of origin. If you look out to sea on a calm day, the waves that you see will be swell waves from a distant storm.

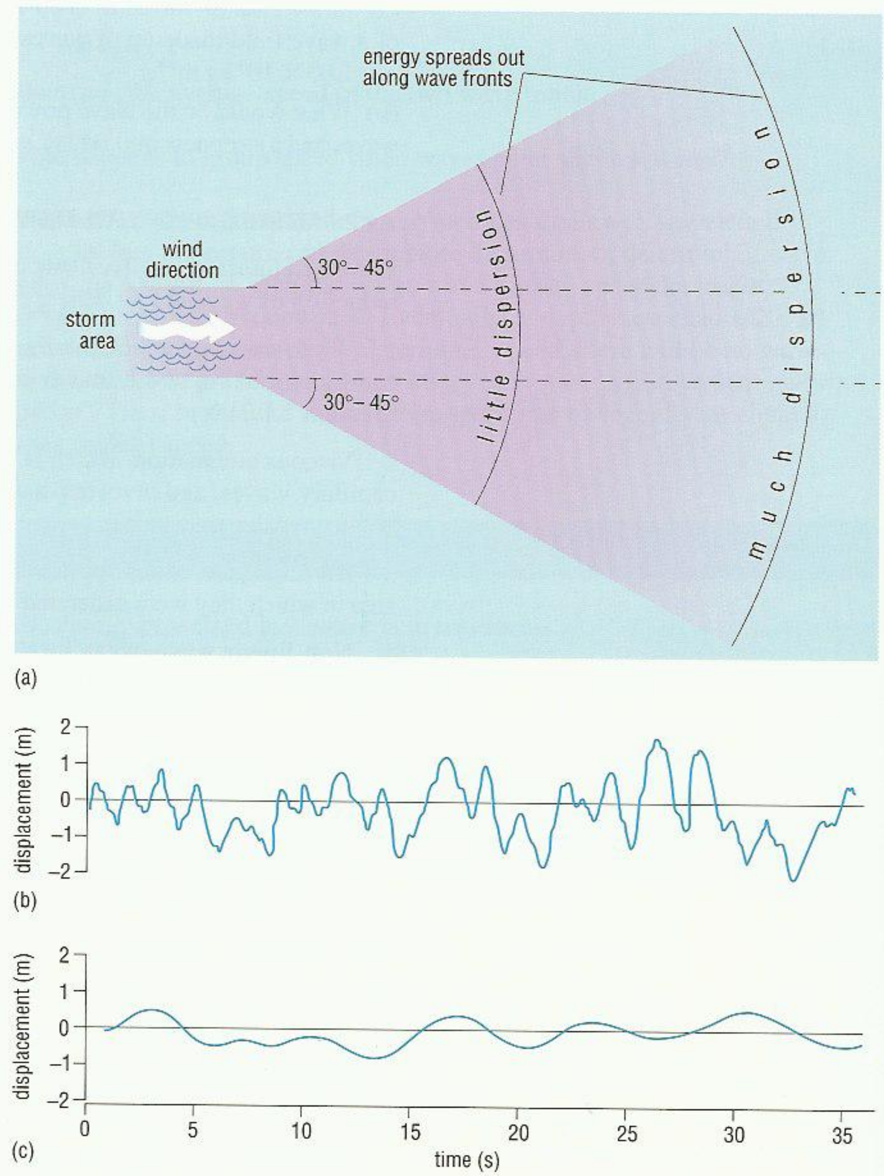


Figure 1.11 (a) The spreading of swell from a storm centre, showing the area in which swell might be expected. As distance from the storm increases, the length of the wave crest increases, with a corresponding decrease in wave height and energy per unit length of wave – spreading loss, discussed opposite.
(b) Wave record near a storm centre.
(c) Wave record of swell, well away from the storm centre.

Systematic observations show that local winds and waves have very little effect on the size and progress of swell waves, and swell seems able to pass through locally generated seas without hindrance or interaction. Once swell waves have left the storm area, their wave height gradually diminishes, chiefly because the wave crests lengthen over a progressively wider front (Figure 1.11(a)) but also because of energy loss caused by air resistance to steep waves (item 3 of Section 1.4.2). Once wave height has diminished to a few tens of centimetres, swell waves are not steep enough to be significantly influenced by local winds.

In the ocean, we find waves travelling in many directions, resulting in a confused sea. To achieve a complete description of such a sea-surface, the amplitude, frequency and direction of travel of each component are needed. The energy distribution of the sea-surface (cf. Figure 1.4) can then be calculated, but, as you might imagine, such a complex process requires expensive equipment to measure the wave characteristics, and computer facilities to perform the necessary calculations.

One or more components of a confused sea may be long waves or swell resulting from distant storms. In practice, about 90% of the sea-surface energy generated by the storm propagates within an angle of 30° to 45° either side of the wind direction. Consequently, waves generated by a storm in a localized region of a large ocean radiate outwards as a segment of a circle (Figure 1.11(a)). As the circumference of the circle increases, the energy per unit length of wave crest must decrease (and so must wave height), so that the total energy of the wave front remains the same. This decrease in energy per unit length of wave crest is known as **spreading loss** (of wave energy), and in the case of established swell waves there is very little loss of wave energy apart from that caused by spreading over a progressively wider front.

The waves with the longest periods travel fastest, and progressively out-distance waves of higher frequencies (shorter periods). Near to the storm, dispersion is likely to be minimal (Figure 1.11(b)), but the further one moves from the storm location, the more clearly separated waves of differing frequencies become, resulting in the regular wave motions we know as swell (Figure 1.11(c)).

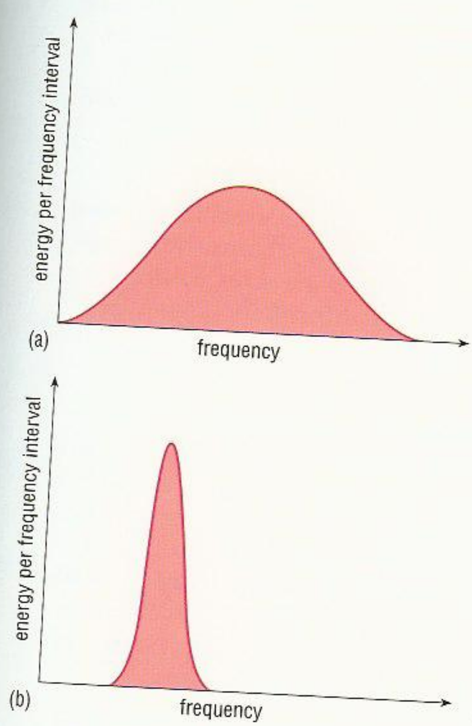


Figure 1.12 Wave-energy spectra, each determined from wave heights measured over a short time interval, for two areas (a) and (b) in the same ocean (not to scale). One area is a storm centre, and the other is far away from the storm. (For use with Question 1.13.)

QUESTION 1.13 Figure 1.12 shows two wave-energy spectra, (a) and (b) (cf. Figure 1.4). One represents the wave field energy in a storm-generating area; and the other represents the energy of the wave field in an area far away from the storm, but receiving swell from it. Which of the two spectra represents which situation?

If you recorded the waves arriving from a storm a great distance (over 1000 km) away, you would, as time progressed, see the peak in the wave-energy spectrum move progressively towards higher frequencies (i.e. shorter periods). By recording the frequencies of each of a series of swell waves arriving at a point, it would be possible to calculate each of their speeds. From the set of speeds, a graph could be plotted to estimate the time and place of their origin. Before the days of meteorological satellites, this method was often used to pinpoint where and when storms had occurred in remote parts of the oceans.

1.5 WAVES APPROACHING THE SHORE

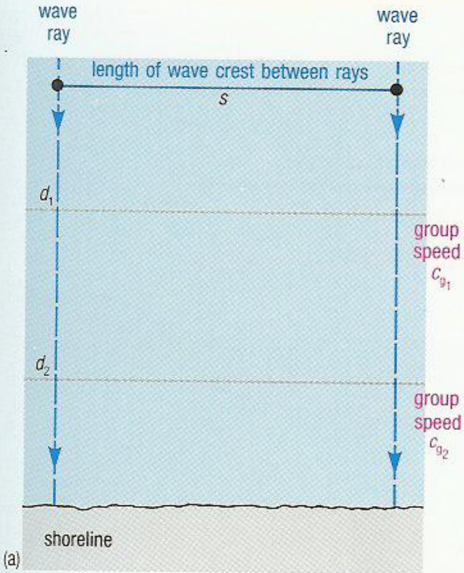


Figure 1.13 Plan view illustrating changes in the speed of waves approaching the shore. Grey lines represent wave crests at depths d_1 and d_2. Wave rays (dashed blue lines) are at right angles (i.e. normal) to the wave crests. For further explanation, see text.

It is a matter of common observation that waves coming onto a beach increase in height and steepness and eventually break. Figure 1.13 shows a length of wave crest, *s*, which is directly approaching a beach. As the water is shoaling, the wave crest passes a first point where the water depth (d_1) is greater than at a second point nearer the shore (where the depth is d_2). We assume that the amount of energy within this length of wave crest remains constant, the wave is not yet ready to break, and that water depth is less than 1/20 of the wavelength (i.e. Equation 1.4 applies: $c = \sqrt{gd}$). Because wave speed in shallow water is related to depth, the speed c_1 at depth d_1 is greater than the speed c_2 at depth d_2. If energy remains constant per unit length of wave crest, then

$$E_1 c_1 s = E_2 c_2 s$$

$$\text{or} \quad \frac{E_2}{E_1} = \frac{c_1}{c_2} \qquad (1.12)$$

and because energy is proportional to the square of the wave height (Equation 1.11) then we can write

$$\frac{E_2}{E_1} = \frac{c_1}{c_2} = \frac{H_2^{\,2}}{H_1^{\,2}} \qquad (1.13)$$

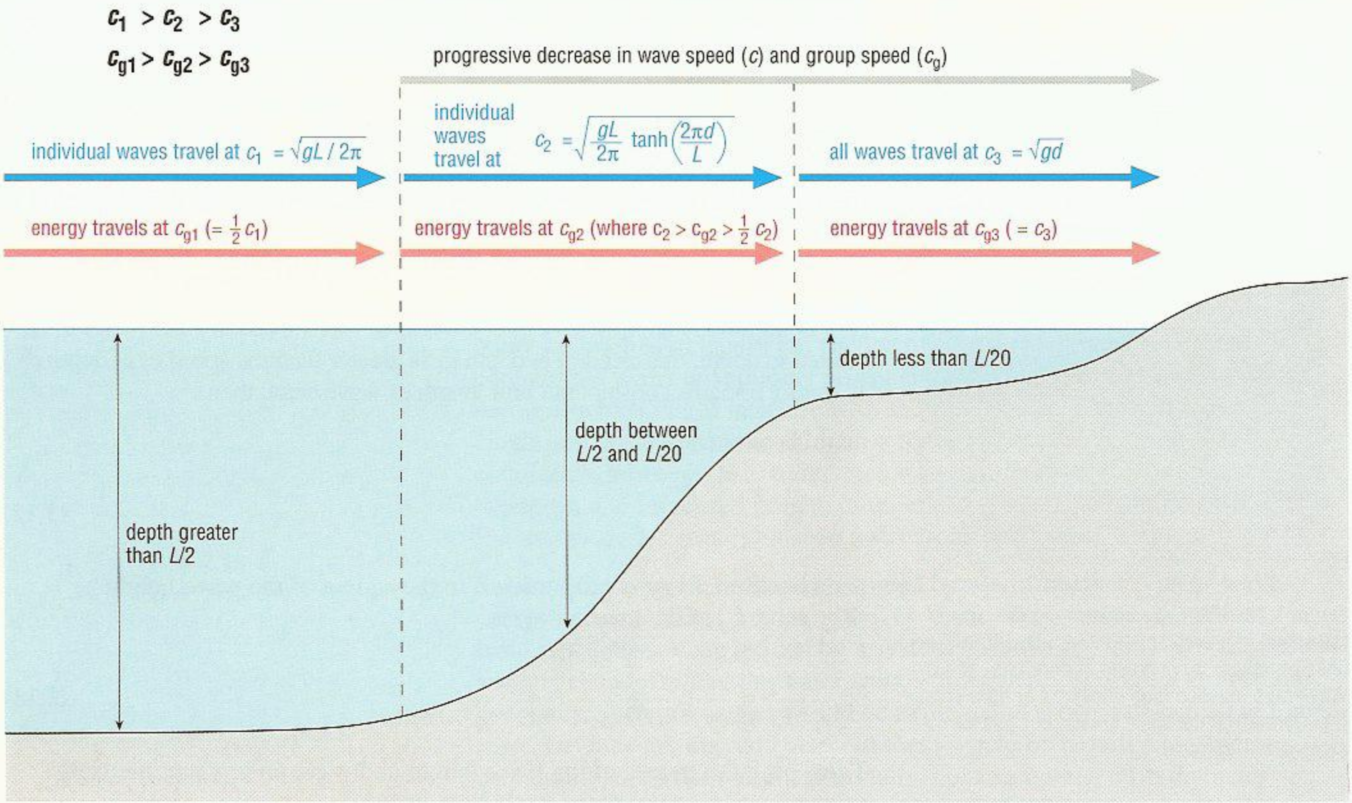
Thus, both the square of the wave height and wave energy are inversely proportional to wave speed in shallow water.

This relationship is straightforward once the wave has entered shallow water. But what happens during the transition from deep to shallow water?

This is quite a difficult question, best answered by considering the highly simplified case illustrated in Figure 1.14. Imagine waves travelling shoreward over deep water (depth greater than half the wavelength). Wave speed is then governed solely by wavelength (Equation 1.3, $c = \sqrt{gL/2\pi}$. The energy is being propagated at the group speed (c_g) which is approximately half the wave speed (c), Section 1.3. As the waves move into shallower water, wave speed becomes governed by both depth and wavelength (Equation 1.2), but once the waves have moved into shallow water, where $d < L/20$, wave speed becomes governed solely by depth (Equation 1.4) and is much reduced. Remember from Section 1.3 that in shallow water group speed is equal to wave speed. The rate at which energy arrives from offshore (Figure 1.14, overleaf) must be equal to the rate at which energy moves inshore; so if the group speed in shallow water is less than half the original wave speed (and hence less than the original group speed) in deep water, the waves will show corresponding increases in height and in energy per unit area.

However, it is essential to realize that while the energy and height of individual waves will increase as they enter shallow water, *the rate of supply of wave energy* (wave power, Section 1.4.1) *must remain constant* (ignoring frictional losses).

As mentioned earlier, when waves move into shallow water, the waves begin to 'feel the bottom', the circular orbits of the water particles become flattened (Figure 1.8(c) and (d)), and wave energy will be dissipated by friction at the sea-bed, resulting in to-and-fro movement of sediments. The gentler the slope of the immediate offshore region, the sooner the incoming waves will 'feel' the bottom, the greater will be the friction with the sea-bed and the greater the energy loss before the waves finally break (see Section 1.5.2).



$c_1 > c_2 > c_3$
$c_{g1} > c_{g2} > c_{g3}$
progressive decrease in wave speed (c) and group speed (c_g)
individual waves travel at $c_1 = \sqrt{gL/2\pi}$
individual waves travel at $c_2 = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)}$
all waves travel at $c_3 = \sqrt{gd}$
energy travels at c_{g1} $(= \frac{1}{2} c_1)$
energy travels at c_{g2} (where $c_2 > c_{g2} > \frac{1}{2} c_2$)
energy travels at c_{g3} $(= c_3)$
depth greater than $L/2$
depth between $L/2$ and $L/20$
depth less than $L/20$

We can estimate increase or decrease in wave size by measuring the distances between wave rays, and applying Equation 1.15. This method is quite useful provided wave rays neither approach each other too closely nor cross over, as in these cases the waves become high, steep and unstable, and simple wave theory becomes inadequate.

1.5.2 WAVES BREAKING UPON THE SHORE

As a wave breaks upon the shore, the energy it received from the wind is dissipated. Some energy is reflected back out to sea, the amount depending upon the slope of the beach – the shallower the angle of the beach slope, the less energy is reflected. Most of the energy is dissipated as heat and sound (the 'roar' of the surf) in the final small-scale mixing of foaming water, sand and shingle. Some energy is used in fracturing large rock or mineral particles into smaller ones, and yet more may be used to move sediments and increase the height and hence the potential energy of the beach form. This last aspect depends upon the type of waves. Small gentle waves and swell tend to build up beaches, whereas storm waves tear them down (see also Chapter 5).

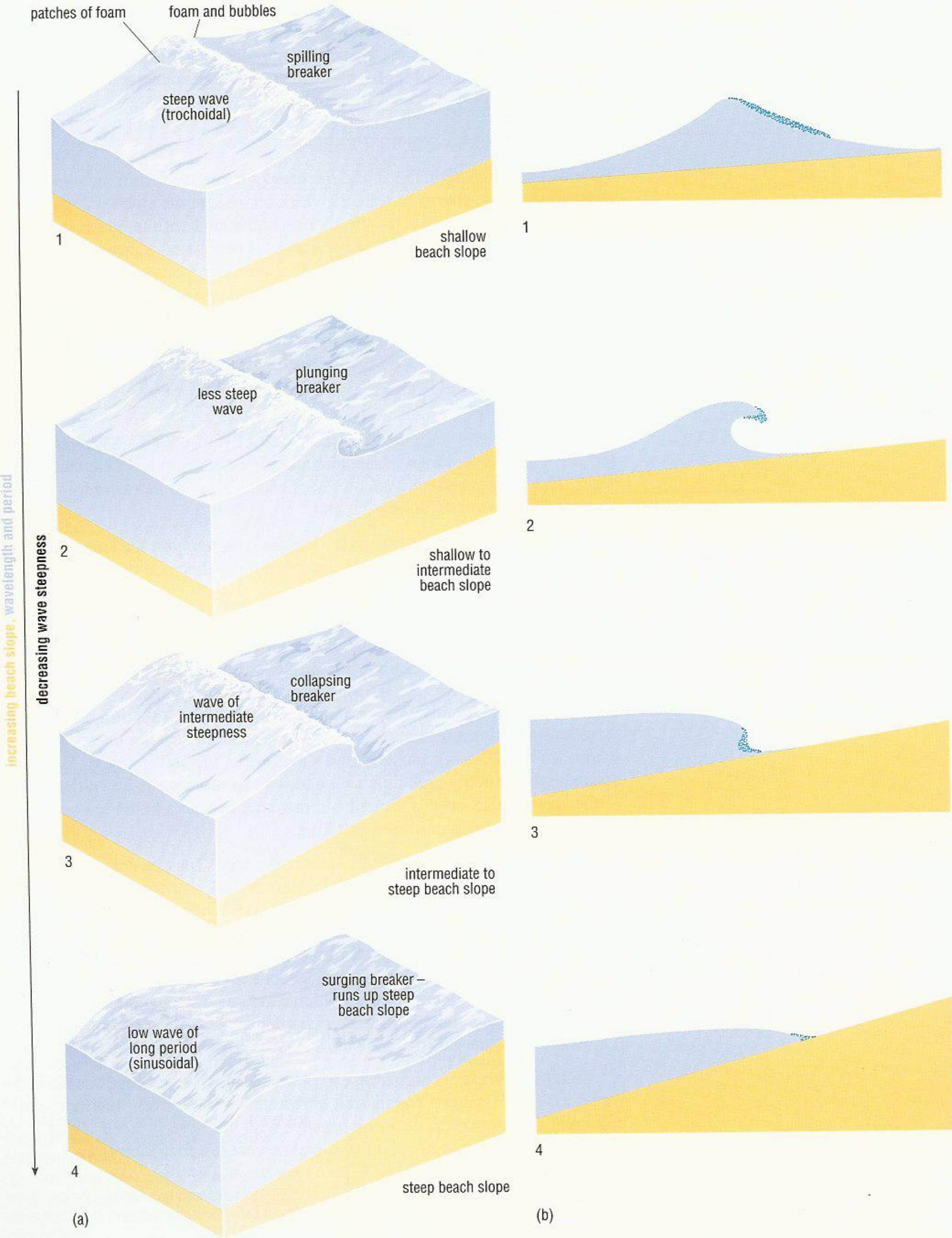
A breaking wave is a highly complex system. Even some distance before the wave breaks, its shape is substantially distorted from a simple sinusoidal wave. Hence the mathematical model of such a wave is more complicated than we have assumed in this Chapter.

Four major types of breaker can be identified, though you may often see breakers of intermediate character and/or of more than one type on the same beach at the same time.

1 Spilling breakers are characterized by foam and turbulence at the wave crest. Spilling usually starts some distance from shore and is caused when a layer of water at the crest moves forward faster than the wave as a whole. Foam eventually covers the leading face of the wave, and such waves are characteristic of a gently sloping shoreline. A tidal bore (Section 2.4.3) is an extreme form of a spilling breaker. Breakers seen on beaches *during* a storm, when the waves are steep and short, are of the spilling type. They dissipate their energy gradually as the top of the wave spills down the front of the crest, which gives a violent and formidable aspect to the sea because of the more extended period of breaking.

2 Plunging breakers are the most spectacular type. The classical form, much beloved by surf-riders, is arched, with a convex back and concave front. The crest curls over and plunges downwards with considerable force, dissipating its energy over a short distance. Plunging breakers on beaches of relatively gentle slope are usually associated with the long swells generated by distant storms. Locally generated storm waves seldom develop into plunging breakers on gently sloping beaches, but may do so on steeper ones. The energy dissipated by plunging breakers is concentrated at the *plunge point* (i.e. where the water hits the bed) and can have great erosive effect.

3 Collapsing breakers are similar to plunging breakers, except that the waves may be less steep and instead of the crest curling over, the front face collapses. Such breakers occur on beaches with moderately steep slopes, and under moderate wind conditions, and represent a transition from plunging to surging breakers.


patches of foam
foam and bubbles
spilling breaker
steep wave (trochoidal)
1
shallow beach slope
plunging breaker
less steep wave
2
shallow to intermediate beach slope
collapsing breaker
wave of intermediate steepness
3
intermediate to steep beach slope
surging breaker – runs up steep beach slope
low wave of long period (sinusoidal)
4
steep beach slope
increasing beach slope, wavelength and period
decreasing wave steepness
(a)
1
2
3
4
(b)

4 Surging breakers are found on the very steepest beaches. Surging breakers are typically formed from long, low waves, and the front faces and crests thus remain relatively unbroken as the waves slide up the beach.

Figure 1.18 illustrates the relationship between wave steepness, beach steepness and breaker type.

The way breaker shape changes from top to bottom of the picture depends upon:

1 Increasing beach slope (if considered independently from wave characteristics).

2 Increasing wavelength and period and correspondingly decreasing wave steepness, if these characteristics are considered independently of beach slope.

It is not always possible to consider 1 and 2 separately, because as you will see in later Chapters, beach slope is partly influenced by prevailing wave type and partly by the particle sizes of the beach sediments, which in turn depend upon the energy of the waves which erode, transport and deposit them.

QUESTION 1.15 If you observed plunging breakers on a beach and walked along towards a region where the beach became steeper, what different types of breaker might you expect to see?

From the descriptions, Figure 1.18, and the answer to Question 1.15, it can be seen that the four types of breaker form a continuous series. The spilling breaker, characteristic of shallow beaches and steep waves (i.e. with short periods and large amplitudes), forms one end of the series. At the other end of the series is the surging breaker, characteristic of steep beaches and of waves with long periods and small amplitudes. For a given beach, the arrival of waves steeper than usual will tend to give a type of breaker nearer the 'spilling' end of the series, whereas calmer weather favours the surging type. The dynamics of collapsing (3) and surging (4) breakers are affected by bottom slope more than those of spilling (1) and plunging (2) breakers. Spilling and plunging breakers can also occur in deep water, partly because the sea-bed is far below and does not affect wave dynamics. Collapsing and surging breakers do not occur in deep water.

(c)

Figure 1.18 (a) The four types of breaker seen in perspective view from top to bottom (1–4): spilling, plunging, collapsing, surging. The vertical arrow shows their relationships to beach slope, wave period, length and steepness.
(b) Cross-sections through the four breaker types.
(c) Photograph of a breaker, part spilling, part plunging. See text for further discussion.

IMPORTANT: When examining Figure 1.18, you need to be aware that the four types of breaker illustrated are just stages in a continuous spectrum; changes from one to another are gradual, not instantaneous.

1.6.2 GIANT WAVES

The cultures of all seafaring nations abound with legends of ships being swamped by gigantic waves, and of sightings of waves of almost unbelievable size.

An early objective method for estimating the height of large waves was t[o] send a seaman to climb the rigging until he could just see the horizon ove[r] the top of the highest waves when the ship was in a wave trough. This technique was used by the *Venus* during her circumnavigation of the worl[d] from 1836 to 1839. She did not meet any particularly high waves during h[er] voyage – the highest estimated by this method was about 8 m high, off Ca[pe] Horn. The highest reliably measured wave was encountered by the US tanker *Ramapo*, *en route* from Manila to San Diego across the North Pacif[ic] in 1933, when she was overtaken by waves having heights up to 34 m.

A region of the ocean that is infamous for encounters with giant waves is the Agulhas Current off the east coast of South Africa (Figure 1.19(a)). Waves travelling north-east from the southern Atlantic Ocean tend to be refracted and focused by the current, and wave rays can be plotted, as outlined in Section 1.5.1, and illustrated diagrammatically in Figure 1.19(b)) and they are further steepened and shortened by the counter-current effect outlined in Section 1.6.1. Wave periods of 14 s, with correspondingly long wavelengths of about 300 m, are quite common. Wave heights in this region can be of the order of 30 m which would result in a very steep wave (steepness = 0.1). Such high and steep waves are sometimes preceded by correspondingly deep troughs, which are particularly dangerous as they can only be seen by vessels that are on the crest of the preceding wave.

Look again at Figure 1.5. The high wave occurring 122 s into the wave record is preceded by a particularly deep trough, but that is not the case for the high wave occurring at 173 s. As we saw in Section 1.1.4, ocean waves are rarely regular, and it is usually not possible to predict the heights of individual waves, nor the depths of individual troughs.

QUESTION 1.16 An elderly ex-seaman, in his cups, claims to have seen gigantic waves in the Southern Ocean, successive peaks of which took 30 seconds to pass, and which had wavelengths twice as long as his ship. Can you believe him?

Before dismissing the sailor's claim in Question 1.16 as a tall story, let us examine it more closely. Let us suppose his ship was travelling in the same direction as the waves and was being *overtaken* by them, and he had made the simple mistake of not taking account of the ship's velocity with respect to the waves when timing the intervals between successive peaks.

QUESTION 1.17 Further conversation with the seaman established that his ship was the cruiser HMS *Exeter* (1929–42), which at the time of the incident was steaming at 23 knots (11.8 m s^{-1}) in the same direction as the waves. Given that the *Exeter* was 575 feet in length (175 m), can you believe him now?

1.6.3 TSUNAMIS

Tsunami is a Japanese word for ocean waves of very great wavelength. Such waves are caused chiefly by seismic disturbances (earthquakes), but also by slumping of unstable masses of submarine sediment or rock (see Chapter 3), and by 'splashdown' of comets or asteroids in the sea. Among the most famous tsunamis is the one triggered by the Lisbon earthquake of 1755, which caused at least as much destruction as the earthquake itself. Although commonly referred to as 'tidal waves', *tsunamis are not related to the tides*. Tsunamis commonly have wavelengths of the order of hundreds of kilometres, so even in the open ocean, the ratio of wavelength to depth is such that a tsunami travels as a shallow-water wave, i.e. its speed is *always* governed by the depth of ocean over which it is passing.

QUESTION 1.18 What would be the speed of a tsunami across the open ocean above an abyssal plain? (Assume the average depth is 5.5 km.)

Although a tsunami travels at great speed in the open ocean, its wave height is small, usually in the order of one metre, and so it often remains undetected. On reaching shallow coastal waters, however, the speed diminishes, while the *power* of the wave remains the same (Section 1.4.1). Hence the wave energy (and therefore the wave height, Equations 1.11 and 1.13) must increase.

Great destruction can be wreaked by a tsunami. It is not unknown for people on board ships at anchor offshore to be unaware of a tsunami passing beneath them, but to witness the adjacent shoreline being pounded by large waves only a few seconds later. Tsunamis occur most frequently in the Pacific, because that ocean experiences frequent seismic activity, particularly around its margins. Accurate earthquake detection can give warning of the approach of tsunamis to coasts some distance from the earthquakes. Around and across the Pacific Ocean is a long-established system of warning stations, of which Honolulu is the administrative and geographical centre.

1.6.4 SEICHES

A **seiche** is a standing wave, which can be considered as the sum of two progressive waves travelling in opposite directions (Section 1.1). Seiches can occur in lakes, and also in bays, estuaries or harbours which are open to the sea at one end. A seiche can be approximately modelled by sliding up and down in the bath, or – perhaps more accurately – with a flat dish containing water, gently moving it to and fro and setting the water into oscillatory motion, as often happens inadvertently when the tray below the freezer compartment of a defrosted refrigerator is removed, and the water starts sloshing back and forth.

In most bays and estuaries, the water is relatively shallow compared with the seiche wavelength (L), and the period of the seiche is determined by the length of the basin and the depth of water in it. Figure 1.20(a, b) (overleaf) shows the idealized water motions in a seiche in a closed basin (e.g. a lake). At either end of the container, water level is alternately high and low, whereas in the middle the water level remains constant. The length of the container (l) corresponds to half the wavelength (L) of the seiche.

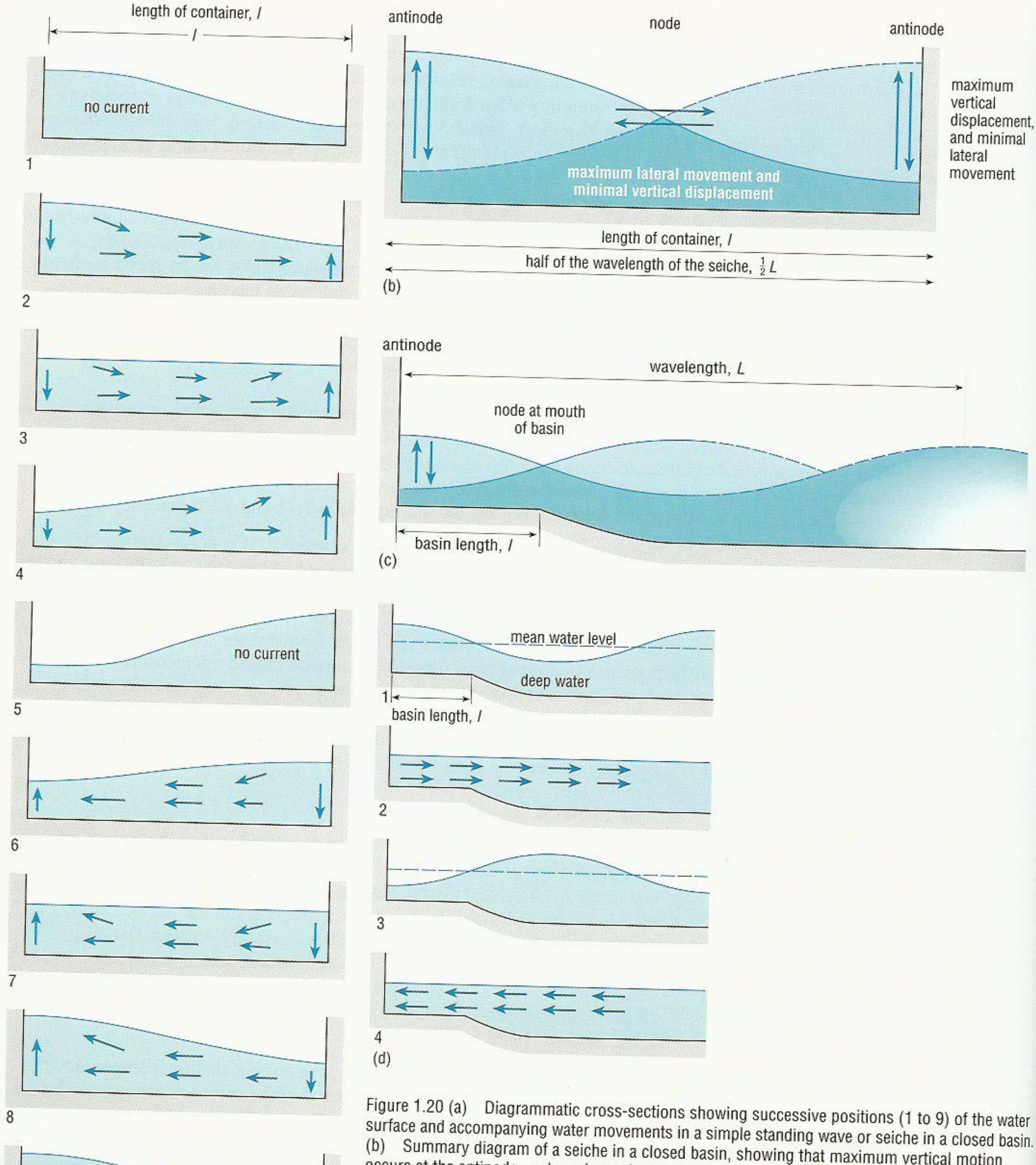


Figure 1.20 (a) Diagrammatic cross-sections showing successive positions (1 to 9) of the water surface and accompanying water movements in a simple standing wave or seiche in a closed basin. (b) Summary diagram of a seiche in a closed basin, showing that maximum vertical motion occurs at the antinode, and maximum horizontal motion occurs at the node. (c) Diagrammatic cross-section of a quarter-wavelength standing wave in a small harbour. (d) Sections showing successive positions of the water surface and accompanying water movements in a seiche in a harbour or small bay, in sequence 1 to 4: water flows out of the basin (2) as the wave crest falls, and flows back in (4) as the wave trough fills.

Where the water level is constant (the **node**), the horizontal flow of water from one end of the container to the other is greatest, as the crest and trough of the seiche alternate at either end of the basin. Where the fluctuation of water level is greatest (the **antinodes**, at either end of the basin), there is minimal horizontal movement of the water. These oscillatory water movements in a seiche in a closed basin are summarized in the idealized cross-section, Figure 1.20(b).

If the water depth divided by the length of the container is less than 0.1, then the waves can be considered to behave as shallow-water waves, with speed $= \sqrt{gd}$, and the period of oscillation, T (in seconds), is given by:

$$T = \frac{2l}{\sqrt{gd}} \tag{1.17}$$

where l = length of container (in metres); d = depth (in metres); and $g = 9.8\ \text{m s}^{-2}$.

In some basins open to the sea at one end, i.e. in some bays and estuaries, it is possible for a node to occur at the entrance to the basin and an antinode at the landward end. Figure 1.20(c) is an idealized vertical section of such a situation, and Figure 1.20(d) shows the corresponding oscillatory water movements. In this case, the length of the basin (l) corresponds to a quarter of the wavelength of the seiche (L). The corresponding equation for the period is therefore:

$$T = \frac{4l}{\sqrt{gd}} \tag{1.18}$$

T is here known as the **resonant period**.

For standing waves to develop, the resonant period of the basin must be equal to the period of the wave motion or to a small whole number of multiples of that period.

QUESTION 1.19 A small harbour, open to the sea at one end, is 90 m long and 10 m deep at high water. What would be the effect of swell waves of period 18 s arriving at the harbour mouth?

Your answer to Question 1.19 is an example of how the arrival of waves of a certain frequency can create problems for moored vessels in small harbours by setting up a standing wave. Just as the seiche in a flat dish will 'slop over' if your standing wave gets too big, so a standing wave in a harbour may dash vessels against the harbour wall, or even throw them ashore. When the standing wave is at the low point of the antinode, there is also the danger of vessels being grounded, thus suffering damage to their hulls.

1.8 SUMMARY OF CHAPTER 1

1 Idealized waves of sinusoidal form have wavelength (length between successive crests), height (vertical difference between trough and crest), steepness (ratio of height to length), amplitude (half the wave height), period (length of time between successive waves passing a fixed point) and frequency (reciprocal of period). Water waves show cyclical variations in water level (displacement), from $-a$ (amplitude) in the trough to $+a$ at the crest. Displacement varies not only in space (one wavelength between successive crests) but also in time (one period between crests at one location). Steeper waves depart from the simple sinusoidal model, and more closely resemble a trochoidal wave form.

2 Waves transfer energy across/through material without significant *overall* motion of the material itself, but individual particles are displaced from, and return to, equilibrium positions as each wave passes. Surface waves occur at interfaces between fluids, either because of relative movement between the fluids, or because the fluids are disturbed by an external force (e.g. wind). Waves occurring at interfaces between oceanic water layers are called internal waves. Wind-generated waves, once initiated, are maintained by surface tension and gravity, although only the latter is significant for water waves over 1.7 cm wavelength.

3 Most sea-surface waves are wind-generated. The stronger the wind, the larger the wave, so variable winds produce a range of wave sizes. A constant wind speed produces a fully developed sea, with waves of $H_{1/3}$ (average height of highest 33% of the waves) characteristic of that wind speed. The Beaufort Scale relates sea state and $H_{1/3}$ to the causative wind speed.

4 Water particles in waves in deep water follow almost circular paths, but with a small net forward drift. Path diameters at the surface correspond to wave heights, but decrease exponentially with depth. In shallow water, the orbits become flattened near the sea-bed. For waves in water deeper than 1/2 wavelength, wave speed equals wavelength/period ($c = L/T$) and is proportional to the square root of the wavelength ($c = \sqrt{gL/2\pi}$); it is unaffected by depth. For waves in water shallower than 1/20 wavelength, wave speed is proportional to the square root of the depth ($c = \sqrt{gd}$) and does not depend upon the wavelength. For idealized water waves, the three characteristics, c, L and T, are related by the equation $c = L/T$. In addition, each can be expressed in terms of each of the other two. For example, $c = 1.56T$ and $L = 1.56T^2$.

5 Waves of different wavelengths become dispersed, because those with greater wavelengths and longer periods travel faster than smaller waves. If two wave trains of similar wavelength and amplitude travel over the same sea area, they interact. Where they are in phase, displacement is doubled, whereas where they are out of phase, displacement is zero. A single wave train results, travelling as a series of wave groups, each separated from adjacent groups by an almost wave-free region. Wave group speed in deep water is half the wave (phase) speed. In shallowing water, wave speed approaches group speed, until the two coincide at depths less than 1/20 of the wavelength, where $c = \sqrt{gd}$.

6 Wave energy is proportional to the square of the wave height, and travels at the group speed. Wave power is rate of supply of wave energy, and so it is wave energy multiplied by wave (or group) speed, i.e. it is wave energy propagated per second per unit length of wave crest (or wave speed multiplied by wave energy per unit area). Total wave power is conserved, so waves entering shallowing water and/or funnelled into a bay or estuary (see also 7 below) increase in height as their group speed falls. Wave energy has been successfully harnessed on a small scale, but large-scale utilization involves environmental and navigational problems, and huge capital outlay.

7 Dissipation of wave energy (attenuation of waves) results from white-capping, friction between water molecules, air resistance, and non-linear wave–wave interaction (exchange of energy between waves of differing frequencies). Most attenuation takes place in and near the storm area. Swell waves are storm-generated waves that have travelled far from their place of origin, and are little affected by wind or by shorter, high-frequency waves. The wave energy associated with a given length of wave crest decreases with increasing distance from the storm, as the wave energy is spread over an ever-increasing length of wave front.

8 Waves in shallow water may be refracted. Variations in depth cause variations in speed of different parts of the wave crest; the resulting refraction causes wave crests to become increasingly parallel with bottom contours. The energy of refracted waves is conserved, so converging waves tend to increase, and diverging waves to diminish, in height. Waves in shallow water dissipate energy by frictional interaction with the sea-bed, and by breaking. In general, the steeper the wave and the shallower the beach, the further offshore dissipation begins. Breakers form a continuous series from steep spilling types to long-period surging breakers.

9 Waves propagating with a current have diminished heights, whereas a counter-current increases wave height, unless current speed exceeds half the wave group speed. If so, waves no longer propagate, but increase in height until they become unstable and break. Tsunamis are caused by earthquakes or by slumping of sediments, and their great wavelength means their speed is always governed by the ocean depth. Wave height is small in the open ocean, but can become destructively large near the shore. Seiches (standing waves) are oscillations of water bodies, such that at antinodes there are great variations of water level but little lateral water movement, whereas at nodes the converse is true. The period of oscillation is proportional to basin length and inversely proportional to the square root of the depth. A seiche is readily established when the wavelength of incoming waves is four times the length of the basin.

10 Waves are measured by a variety of methods, e.g. pressure gauges on the sea-floor, accelerometers in buoys on the sea-surface, and via remote-sensing from satellites.

ANSWERS AND COMMENTS TO QUESTIONS

CHAPTER 1

Question 1.1 Five seconds. The frequency is $0.2\ s^{-1}$, i.e during one second '0.2 of a wave' passes a fixed point. To find out how long it takes for the whole wave to pass (the period), we need to divide 1 by $0.2\ s^{-1}$:

$T = 1/0.2\ s^{-1} = 5\ s$.

Question 1.2 The less steep of the two. Since steepness = H/L, and H is the same for both waves, the less steep wave will have the greater wavelength, and hence travel faster.

Question 1.3 It decreases. Figure 1.4 shows that the higher the average frequency of the wave field, the smaller the area under the curve. The $\sim 20\ m\ s^{-1}$ (40-knot) spectrum contains much more energy than either of the other two spectra. Most of this energy is related to the low frequency (long period) waves that a 40-knot wind would generate.

Question 1.4 Sixteen waves in 64 seconds = a period of 64/16 seconds = 4 s. The frequency is thus the reciprocal of 4 s, i.e. $1/4\ s = 0.25\ s^{-1}$.

Question 1.5 First, convert frequency f to period T. $T = 1/f = 1/0.05 = 20$ s.

(a) $-a$ (a trough at P), because $30\ s = 1\tfrac{1}{2} \times 20\ s$.

(b) $+a$ (a peak at P), because $80\ s = 4 \times 20\ s$.

(c) 0, because $85\ s = 4\tfrac{1}{4} \times 20\ s$ (η changes from $+a$ to $-a$ in 10 seconds, so five seconds after a peak ($+a$), the displacement is zero).

For (d), (e) and (f): the distance between P and Q is half a wavelength. Note that if the displacement at P is zero and is diminishing, then the displacement at Q is zero and is increasing (and *vice versa*). Hence:

(d) 0

(e) $+a$

(f) 0

Question 1.6 If $k = 2\pi/L$, and $\sigma = 2\pi/T$, then $L = 2\pi/k$, and $T = 2\pi/\sigma$. Substituting into $c = L/T$, we have:

$$c = \frac{2\pi / k}{2\pi / \sigma} = \frac{1/k}{1/\sigma} = \frac{\sigma}{k}$$

In basic units, angular frequency/wave number is $s^{-1}/m^{-1} = m\ s^{-1}$, i.e. speed.

Question 1.7 (a) If d is greater than $0.5L$, then $2d$ is greater than L, and the expression $2\pi d/L$, becomes greater than π. The tanh of numbers greater than π approximates to 1. So $\tanh(2\pi d/L) \approx 1$ and Equation 1.2 approximates to:

$$c = \sqrt{\frac{gL}{2\pi}}$$

(b) If d/L is very small, then $2\pi d/L$ is also very small, and hence $\tanh(2\pi d/L)$ approximates to $2\pi d/L$. So Equation 1.2 becomes:

$$c = \sqrt{\frac{gL2\pi d}{2\pi L}}$$
$$= \sqrt{gd}$$

Question 1.8 From Equation 1.1, $c = L/T$.

From Equation 1.3, $c = \sqrt{gL/2\pi}$

So $\sqrt{gL/2\pi} = L/T$, and (squaring both sides)
$gL/2\pi = L^2/T^2$.

From which $L/T^2 = g/2\pi$ and
$L = gT^2/2\pi$.

Question 1.9 (a) $g = 9.8\ \text{m s}^{-2}$, and $\pi = 3.14$, so $g/2\pi = 1.56\ \text{m s}^{-2}$.

(b) In Equation 1.7: $c = \sqrt{1.56\ \text{m s}^{-2} \times \text{m}}$ which gives units of $\sqrt{\text{m}^2\,\text{s}^{-2}} = \text{m s}^{-1}$.

In Equation 1.8: $L = 1.56\ \text{m s}^{-2} \times \text{s}^2$, which gives units of m (s^{-2} and s^2 cancel out).

In Equation 1.9: $c = 1.56\ \text{m s}^{-2} \times \text{s}$, which gives units of m s^{-1} ($\text{s}^{-2} \times \text{s} = \text{s}^{-1}$).

Question 1.10 (a) $31.2\ \text{m s}^{-2}$. You may have done this the hard way by $L = 1.56 \times 20 \times 20 = 624$, followed by $c = 624/20 = 31.2\ \text{m s}^{-2}$. Better still, you may have used Equation 1.9, and done the sum in one step.

(b) The deep water speed of the wave will be $22.1\ \text{m s}^{-1}$. From Equation 1.7,

$$c = \sqrt{1.56 \times 312} = \sqrt{486.7}$$
$$= 22.1\ \text{m s}^{-1}.$$

(c) The answer is $10.8\ \text{m s}^{-1}$ in *both* cases. If the depth is less than 1/20 of the wavelength, all waves will travel at the same depth-determined speed, i.e. the depth is the only controlling factor. So from Equation 1.4, we get:

$$c = \sqrt{gd}$$
$$= \sqrt{9.8 \times 12} = \sqrt{117.6}$$
$$= 10.8\ \text{m s}^{-1}$$

Question 1.11 No. It would be quadrupled, because the energy of a wave varies with the square of the wave height (Equation 1.11), and hence with the square of the wave amplitude.

Question 1.12 (a) If amplitude is 1.3 m, then wave height is 2.6 m. The values of the constants g and ρ, and also the above value for wave height, can be plugged into Equation 1.11, giving:

$$E = \tfrac{1}{8} \times 1.03 \times 10^3 \times 9.8 \times 2.6^2$$
$$= 8.5 \times 10^3\ \text{J m}^{-2}$$

(The units work out as: $\text{kg m}^{-3} \times \text{m s}^{-1} \times \text{m}^2 = \text{kg s}^{-2}$;
$\text{J} = \text{kg m}^2\,\text{s}^{-2}$, so J m^{-2} is $\text{kg m}^2\,\text{s}^{-2}\,\text{m}^{-2} = \text{kg s}^{-2}$.)

(b) The wave power per unit length is the product of the wave energy per unit area and the group speed. We know the wave energy from (a) above, and can calculate the group speed from the height and steepness as follows:

steepness (0.04) = height (2.6 m) / wavelength.

So wavelength = 2.6/0.04 = 65 m.

From Equation 1.3, wave speed, $c = \sqrt{gl/2\pi}$

So $c = \sqrt{1.56L}$ (Eqn 1.7)

$= \sqrt{1.56 \times 65}$

$\approx 10\,\mathrm{m\,s^{-1}}$

From which group speed, $c_g = 10/2 = 5\,\mathrm{m\,s^{-1}}$.

So wave power = $8.5 \times 10^3\,\mathrm{J\,m^{-2}} \times 5\,\mathrm{m\,s^{-1}} = 42.5\,\mathrm{kW\,m^{-1}}$.

Question 1.13 Spectrum (a) of Figure 1.12 shows the wave energy distributed amongst a wide range of frequencies. The peak is rather poorly defined, and hence must represent the storm-generating area. Spectrum (b), on the other hand, has a much narrower range of frequencies, and a clearly defined peak which has shifted to lower frequencies. It thus represents the regular swell waves at a point well away from the storm. The total energy of spectrum (b), as represented by the coloured area under the curve, is smaller than in (a), because the waves have lost some of their energy in transit, as outlined in the text.

Question 1.14 The wave refraction diagram (Figure 1.17(b)) illustrates how the offshore Hudson Canyon is effective in defocusing storm waves as they approach the Long Branch coastal section from the east–south-east, and in refracting them onto other beaches. Fishermen can leave their boats on the beach at Long Branch during all seasons of the year, despite its apparent exposure to the full force of Atlantic storms. The wave rays are, if anything, focused as they enter the mouth of the Hudson River, so that the energy of storm waves would certainly not be diminished there, and might even be increased. People leaving their boats in this apparently sheltered region could thus be courting disaster.

Question 1.15 If the change in the beach slope was sufficient, you might expect to see collapsing breakers, and if it got really steep, surging breakers as well.

Question 1.16 You have no information on the length of the ship, but if you calculate the wavelength corresponding to a period of 30 s, using Equation 1.8, you get $L = 1.56T^2 = 1.56 \times 900 = 1404$ m. You might conclude that the sailor is trying to tell you that his ship was about 700 m long (nearly half a mile). The longest of today's supertankers are only about 320 m. However, return to the main text and read on …

Question 1.17 *Exeter* was 175 m in length, so if the seaman's story were true, the wavelengths concerned were 350 m. The ship was travelling at $11.8\,\mathrm{m\,s^{-1}}$, so in 30 seconds it would have travelled 30×11.8 m = 354 m. In 30 seconds, an overtaking wave would have travelled one wavelength plus the distance the ship had travelled, i.e. 354 + 350 = 704 m, and the wave speed would be $704/30 = 23.5\,\mathrm{m\,s^{-1}}$.

From Equations 1.3 and 1.7, we can find the wave speed corresponding to a wavelength of 350 m, i.e.

$$c = \sqrt{gL/2\pi} = \sqrt{1.56L}$$

$$= \sqrt{1.56 \times 350} = 23.4\ \mathrm{m\,s^{-1}}$$

which means the sailor's tale is at least consistent with simple wave theory. Full marks if you suspected something of this sort while attempting Question 1.16.

Question 1.18 Because the wavelength is very long compared with an ocean depth of 5500 m over the abyssal plains, the tsunami must be treated as a shallow-water wave (Equation 1.4):

$$c = \sqrt{9.8 \times 5500} = 232\ \mathrm{m\,s^{-1}},$$

which is more than 800 kilometres per hour!

Question 1.19 In Equation 1.18, $l = 90$ m, $d = 10$ m.

So $T = 4 \times 90 / \sqrt{9.8 \times 10} = 36.36$ seconds. Because the resonant period of the harbour is close to 36 s, waves of period 18 s (half of 36 s) would set up a standing wave in the harbour.