2.3 THE DYNAMIC THEORY OF TIDES

When Newton formulated the equilibrium theory of tides in the seventeenth century, he was conscious that it was only a static treatment of the problem and thus only a rough approximation. He was well aware of discrepancies between the predicted equilibrium tides and the observed tides, but did not pursue the matter any further. The equilibrium theory is of limited practical value, even though certain of its predictions are correct, notably that spring and neap tides will occur at new and full Moon (Figure 2.12), that the range of spring tides will typically be two or three times that of neap tides (cf. Figure 2.1), and that tidal inequality is related to declination (Figure 2.9).

There are a number of reasons why actual tides do not behave as equilibrium tides.

1 As discussed earlier, the wavelength of tidal waves is long relative to depth in the oceans, so *they travel as shallow-water waves* (Section 2.1) and as we have seen (Question 2.2) their speed is governed by $c = \sqrt{gd}$ (Equation 1.4). The speed of any wave longer than a few km is therefore limited to about 230 m s^{-1} in the open ocean, less in shallower seas. This is much slower than the linear velocity of the surface of the rotating Earth with respect to the Moon: 448 m s^{-1} at the Equator (Question 2.2(a)). (In fact, this linear velocity decreases with distance from the Equator: to *c*. 230 m s^{-1} at about latitudes 60° N and 60° S, to 78 m s^{-1} at 80° latitude, and zero at the poles themselves.)

2 In any case, the Earth rotates on its axis far too rapidly for either the inertia of the water masses or the frictional forces at the sea-bed to be overcome fast enough for an equilibrium tide to occur. A time-lag in the oceans' response to the tractive forces is thus inevitable, i.e. there is a tidal *lag*, such that high tide commonly arrives some hours after the passage of the Moon overhead. Because the linear velocity of the surface of the Earth with respect to the Moon decreases polewards (cf. (1) above), the tidal lag is greatest at low latitudes (*c*. 6 hours), decreasing to zero at about latitude 65° – but the precise lag is always constant for a particular location. In addition, at most localities, spring tides occur a day or two after both full and new Moon (cf. Figure 2.12), and the time difference (in days) between the meridian (overhead) passage of full or new Moon and the occurrence of the highest spring high tide is sometimes called the *age of the tide*.

3 The presence of land masses prevents the tidal bulges from directly circumnavigating the globe, and the shape of the ocean basins constrains the direction of tidal flows. In fact, the only region of the oceans where a westward-moving tidal bulge could travel unimpeded around the world is the Southern Ocean surrounding Antarctica.

4 Except at the Equator, all lateral (horizontal) water movements (including tidal currents) are subject to the **Coriolis force**, which deflects winds and currents *cum sole* (literally 'with the Sun'), i.e. to the right, or clockwise, in the Northern Hemisphere, and to the left, or anticlockwise, in the Southern Hemisphere.

The **dynamic theory of tides** was developed during the eighteenth century by scientists and mathematicians such as Bernoulli, Euler and Laplace. They attempted to understand tides by considering ways in which the depths and configurations of the ocean basins, the Coriolis force, inertia, and frictional forces might influence the behaviour of fluids subjected to rhythmic forces resulting from the orbital relationships of Earth, Moon and Sun.

As a consequence of the many and varied factors involved, the dynamic theory of tides is intricate, and solutions of the equations are complex. Nevertheless, the dynamic theory has been steadily refined, and computed theoretical tides are very close approximations to the observed tides.

The combined constraint of ocean basin geometry and the influence of the Coriolis force (items 2 and 4 on p. 66) results in the development of **amphidromic systems**, in each of which the crest of the tidal wave at high water circulates around an **amphidromic point** once during each tidal period (Figures 2.14 and 2.15 overleaf). The tidal range is zero at each amphidromic point, and increases outwards away from it.

In each amphidromic system, **co-tidal lines** can be defined, which link all the points where the tide is at the same stage (or phase) of its cycle. The successive co-tidal lines radiating outwards from the amphidromic point thus indicate the passage of the tidal wave crest around it.

Cutting across co-tidal lines, approximately at right angles to them, are **co-range lines**, which join places having the same tidal range. Co-range lines form more-or-less concentric circles about the amphidromic point, representing larger and larger tidal ranges the further away they are from it. Figure 2.14 shows the amphidromic systems for the North Sea, and Figure 2.15 shows the computed world-wide amphidromic systems for the dominant tidal component resulting from the diurnal influence of the Moon (see also Section 2.3.1).

QUESTION 2.6

(a) Assume that a high tide coincides with the co-tidal lines marked zero (i.e. '0') on Figure 2.14. At what stage of the tidal cycle is:

1 The Wash?

2 The Firth of Forth?

(b) Which of (1) and (2) has the greater tidal range?

Inspection of Figures 2.14 and 2.15 shows that, with a few exceptions, the tidal waves of amphidromic systems tend to rotate anticlockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere. At first sight, this pattern of rotation appears to conflict with the principle that the Coriolis force deflects moving fluid masses *cum sole*, but we need to bear in mind that the direction of motion of tidal *waves* is not synonymous with the movement of individual parcels of water.

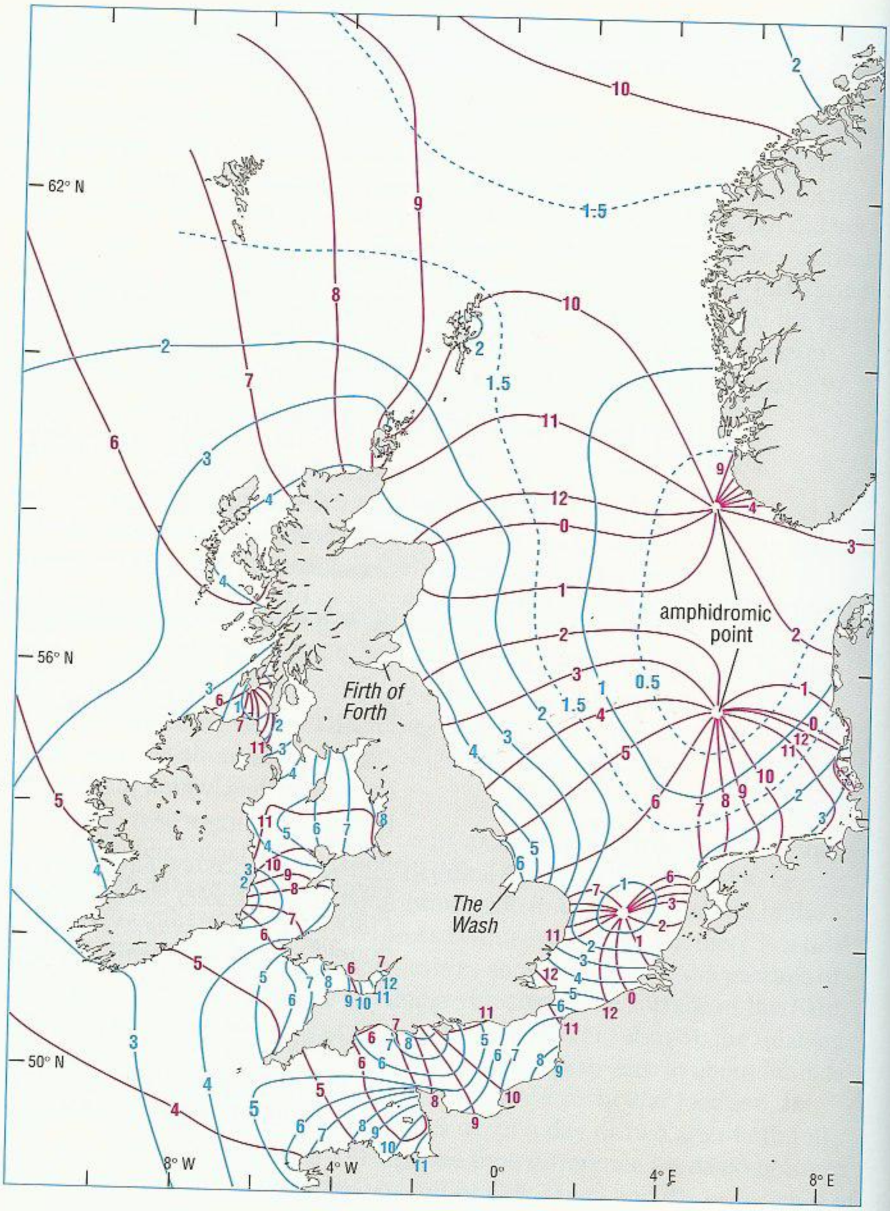


Figure 2.14 Amphidromic systems around the British Isles. The figures on the co-tidal lines (red) indicate the time of high water (in hours) after the Moon has passed the Greenwich meridian. Blue lines are co-range lines, with tidal range in metres.

Consider the enclosed basin shown in Figure 2.16 (on p. 70). The 'bent' arrows in Figure 2.16(a) show how water moving in response to the flooding tide, i.e. in the tidal currents, is deflected to the right by the Coriolis force (the basin is in the Northern Hemisphere), and the water is piled up on the eastern side. Conversely, when the tide ebbs, the water becomes piled up on the western side (Figure 2.16(b)). Hence, because the tidal wave is constrained by land masses, an *anticlockwise* amphidromic system is set up (Figure 2.16(c) and (d)).

It is also very important to remember that tidal waves behave as shallow-water waves, so their orbital motions are flattened like those in Figure 1.8(d). Tidal currents are the horizontal water movements that accompany the rise and fall of the tides as the tidal wave *form* rotates about the amphidromic point, and of course tidal currents change direction during the tidal cycle (see Section 2.4.1).

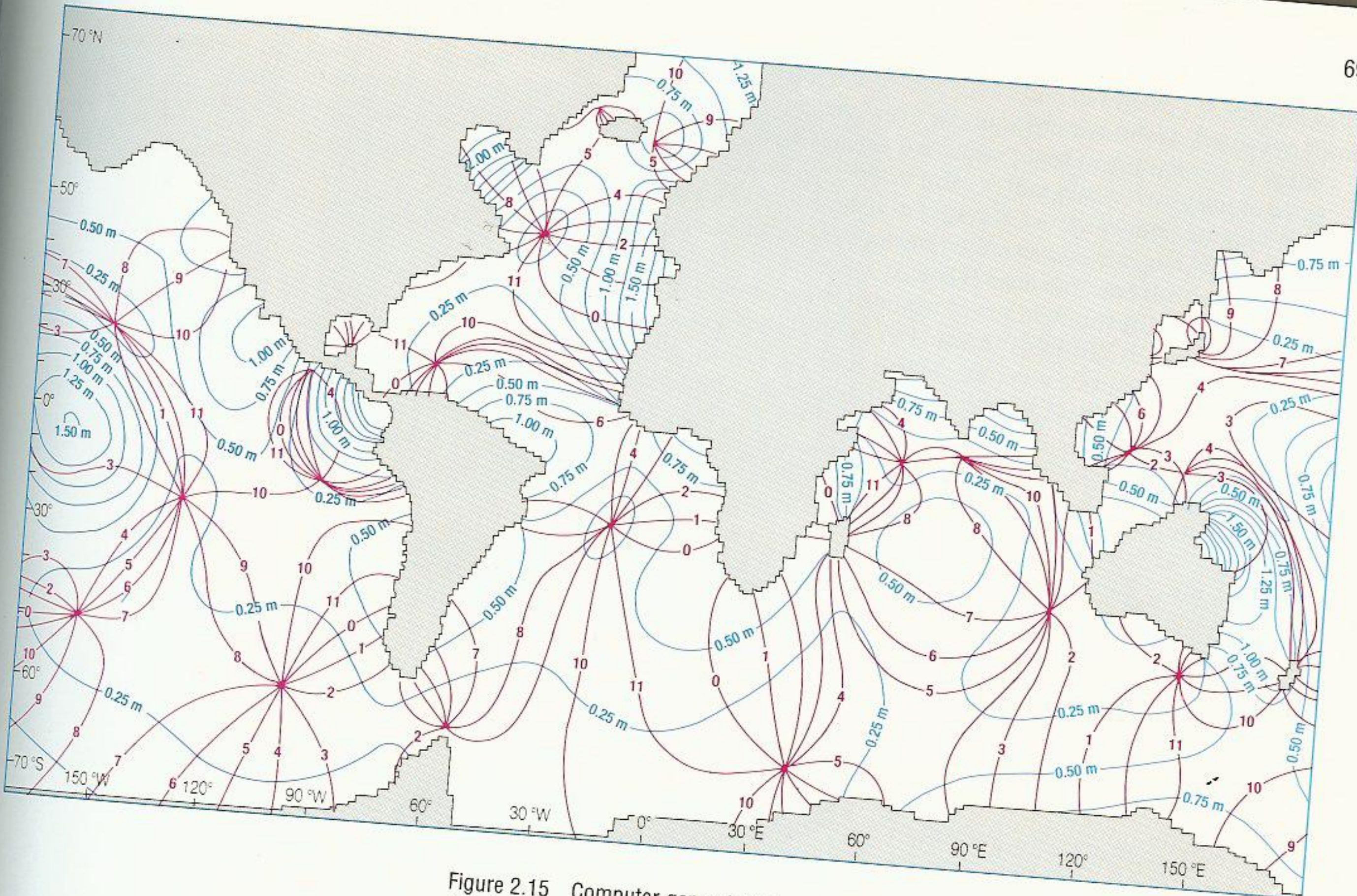


Figure 2.15 Computer-generated diagram of world-wide amphidromic systems for the dominant semi-diurnal lunar tidal component M_2 (see Table 2.1 on p. 71). Blue lines are co-range lines and red lines are co-tidal lines.

The main exceptions to the general pattern of rotation of tidal waves round amphidromic points shown on Figure 2.15 are amphidromic systems less obviously constrained by land masses, e.g. in the South Atlantic (centred on 20° S, 15° W), mid-Pacific (centred on 20° S, 130° W), and North Pacific (centred on 25° N, 155° W); or in certain cases where the amphidromic system rotates about an island, e.g. Madagascar.

QUESTION 2.7

(a) Locate on Figure 2.15 the amphidromic systems identified above, and state how they are exceptions to the general pattern.

(b) Locate the amphidromic system centred near 65° E, 5° N in the north-west Indian Ocean. In what way is this also anomalous?

Tidal waves in amphidromic systems are a type of **Kelvin wave**, in which the amplitude is greatest near coasts (Figure 2.16). Kelvin waves occur where the deflection caused by the Coriolis force is either constrained (as at coasts) or is zero (as at the Equator).

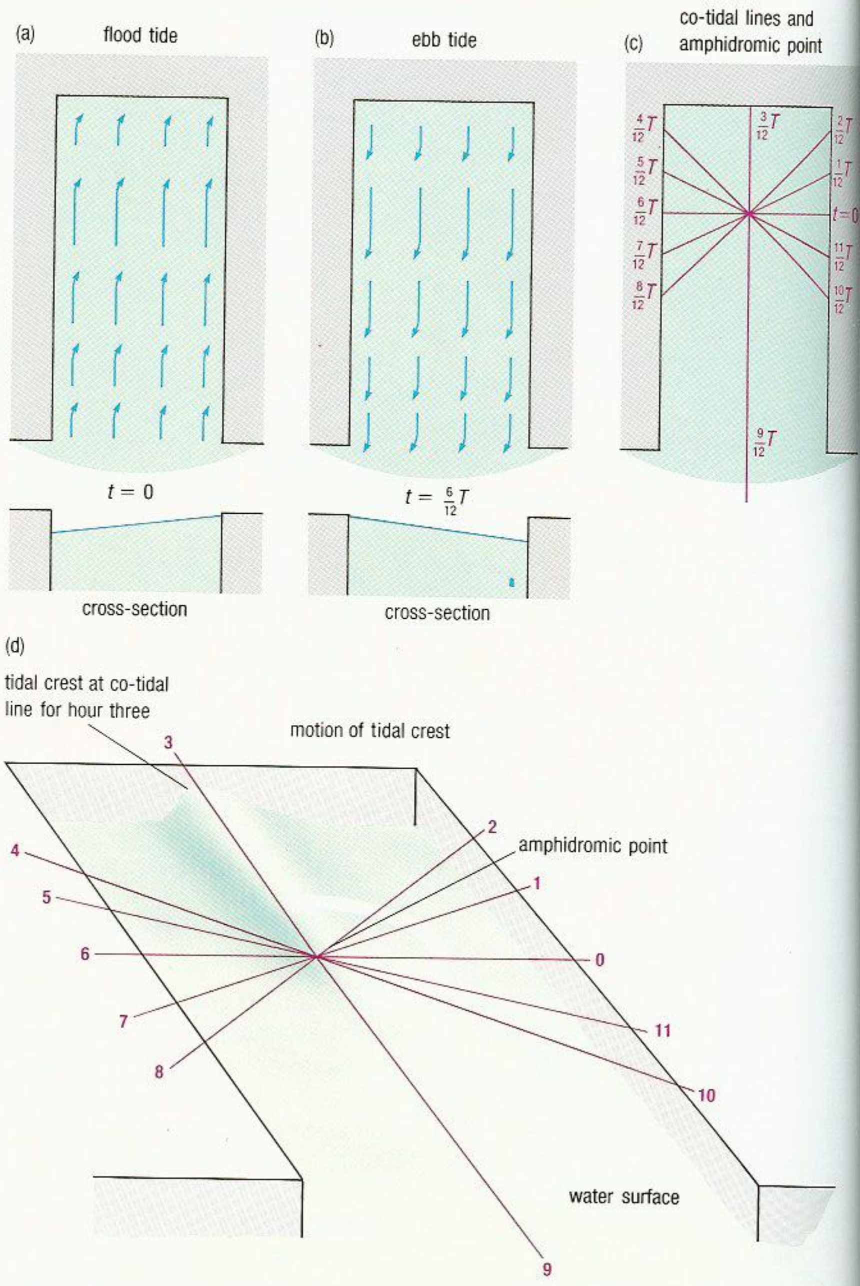


Figure 2.16 The development of an amphidromic system. The hypothetical basin shown is in the Northern Hemisphere.
Note: In (a) and (b), the 'bent' arrows show how the Coriolis force deflects the moving water, 'piling it up' against the sides of the basin.
(a) Flood tide. Water is deflected by the Coriolis force to the right, i.e. towards the east.
(b) Ebb tide. Returning water is deflected by the Coriolis force to the right, i.e. towards the west.
(c) An anticlockwise amphidromic system is established. Times, t, are in twelfth parts of the tidal period, T (= 12 hr 25 min.).
(d) The tidal wave travels anticlockwise. Numbers on co-tidal lines correspond to values of t in (c).

2.3.1 PREDICTION OF TIDES BY THE HARMONIC METHOD

The harmonic method is the practical application of the dynamic theory of tides and is the most usual and satisfactory method for the prediction of tidal heights. It makes use of the knowledge that the observed tide is the sum of a number of harmonic constituents or **partial tides**, each of whose periods precisely corresponds with the period of some component of the relative astronomical motions between Earth, Sun and Moon. For any coastal location, each partial tide has a particular amplitude and phase. In this context, phase means the fraction of the partial tidal cycle that has been completed at a given reference time. It depends upon the period of the tide-producing force concerned, and upon the lag (Section 2.3) of the partial tide for that particular location.

The basic concept is analogous to that illustrated in Figure 1.9, though with a great many more component wave motions (partial tides). The wave form that represents the *actual* tide at a particular place (e.g. Figure 2.1) is the resultant or sum of all of the *partial* tides at that place. An example using just two partial tides is illustrated in Figure 2.17: the combination of a diurnal and a semi-diurnal component produces two unequal high tides (H and h) and two unequal low tides (L and l) each day, and the time interval between the higher low tide (l) and the lower high tide (h) is significantly shorter than that between H and l or L and h. Tides like these, characterized by high and low tides of unequal height, are known as *mixed tides*, and are common, for example, along the Pacific coast of North America (see also Figure 2.18).

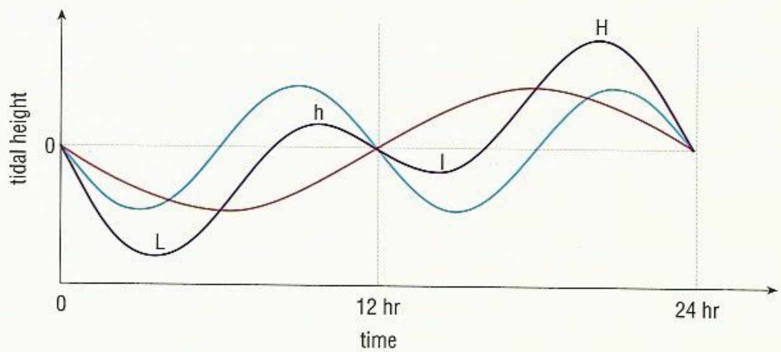


Figure 2.17 Mixed tide (purple) produced by the combination of a diurnal (red) and a semi-diurnal (light blue) partial tidal constituent. H and h = high tides; L and l = low tides. For simplicity, the semi-diurnal period is shown as 12 hr, whereas the M_2 period is 12 hr 25 min.

In order to make accurate tidal predictions for a location such as a seaport, the amplitude and phase for each partial tide that contributes to the actual tide must first be determined from analysis of the observed tides. This requires a record of measured tidal heights obtained over a time that is long compared with the periods of the partial tides concerned. As many as 390 harmonic constituents have been identified. Table 2.1 shows the nine most important of these: four semi-diurnal, three diurnal and two longer-period constituents.

Table 2.1 Some principal tidal constituents. The coefficient ratio (column 4) is the ratio of the amplitude of the tidal component to that of M_2.

Name of tidal component	Symbol	Period in solar hours	Coefficient ratio (M_2 = 100)
Semi-diurnal:			
Principal lunar	M_2	12.42	100
Principal solar	S_2	12.00	46.6
Larger lunar elliptic	N_2	12.66	19.2
Luni-solar	K_2	11.97	12.7
Diurnal:			
Luni-solar	K_1	23.93	58.4
Principal lunar	O_1	25.82	41.5
Principal solar	P_1	24.07	19.4
Longer period:			
Lunar fortnightly	M_f	327.86	17.2
Lunar monthly	M_m	661.30	9.1

Semi-diurnal partial tides result from tide-producing forces that are symmetrically distributed over the Earth's surface with respect to the Sun and Moon, as illustrated in Figures 2.3 and 2.4. M_2 and S_2 are the most important ones, because they control the spring–neap cycle (Figure 2.12). The last column of Table 2.1 shows that S_2 has only 46.6 per cent of the amplitude of M_2, because although the Sun is much more massive than the Moon it is also much further away (Section 2.2).

Diurnal partial tides (Table 2.1) occur because the two symmetrical tidal bulges of Figures 2.5 and 2.6 cannot develop on the real Earth. This is a consequence of the factors summarized in Section 2.3, including inertia, friction with the sea-bed, and the Earth's rotation. Although *in theory* each tidal constituent could develop its own equilibrium (partial) tidal bulges, the tide-producing forces that cause diurnal tides on the real Earth are asymmetrical, being stronger on the side of the Earth closer to the Sun and/or Moon than on the opposite side.

Before moving on, it is worth noting some regularities among other constituents in Table 2.1. The luni-solar diurnal partial tide, K_1, has twice the period of its semi-diurnal counterpart K_2, but has much greater amplitude, while the average of K_1 and P_1 is exactly 24 hours. Small departures of the periods of some semi-diurnal and diurnal constituents (e.g. N_2, P_1) from a simple relationship with those of M_2 and S_2 result mainly from complications related to the orbits of Moon and Earth (Figures 2.10 and 2.11).

With regard to the longer cycles listed in Table 2.1, the lunar fortnightly period (M_f) works out to 13.66 days, almost exactly half the 27.3-day period of the Moon's rotation about the Earth–Moon centre of mass; while the lunar monthly period (M_m) is very close to the perigee–apogee cycle of 27.5 days mentioned in relation to Figure 2.10. There are of course still longer cycles, an obvious example being the 18.6-year period related to precession of the lunar orbit (Figure 2.10); and there are shorter-period constituents as well (see Section 2.4).

Even using the few major constituents in Table 2.1, analysis of tidal records and production of tide-tables for a port for an entire year used to be a very time-consuming activity. In the early years of harmonic analysis, they were computed by hand. The first machine to do the job was invented by Lord Kelvin in 1872. Electronic computers are admirably suited to this repetitive procedure, and tide-tables for individual ports all over the world now take little time to prepare.

The precision achieved by radar altimeters (Section 1.7.1) is such that tidal ranges in the deep oceans can be determined using information on tidal amplitude and phase extracted from the satellite data. Results are in good agreement with predicted values, and are nowadays supplemented by tidal data from the deep-sea pressure gauges mentioned in Section 1.7, placed at strategic locations in the oceans, far from land.

2.4 REAL TIDES

Having examined the theory, let us see how the actual tides behave in different places. Every partial tide has its own set of amphidromic systems, and their amphidromic points do not necessarily coincide.

Suppose you were at a coastal location close to the amphidromic points of both S_2 and M_2, but far from those of O_1 and K_1. Would you expect the tidal period to be predominantly diurnal or semi-diurnal?

It would be predominantly diurnal. The tidal range increases with distance from the amphidromic point (Figure 2.16), so in this case, the tidal range due to the semi-diurnal constituents would be small relative to that due to the diurnal constituents.

Tides can in fact be classified according to the ratio (F) of the sum of the amplitudes of the two main diurnal constituents (K_1 and O_1) to the sum of the amplitudes of the two main semi-diurnal constituents (M_2 and S_2). Some examples are illustrated in Figure 2.18.

QUESTION 2.8

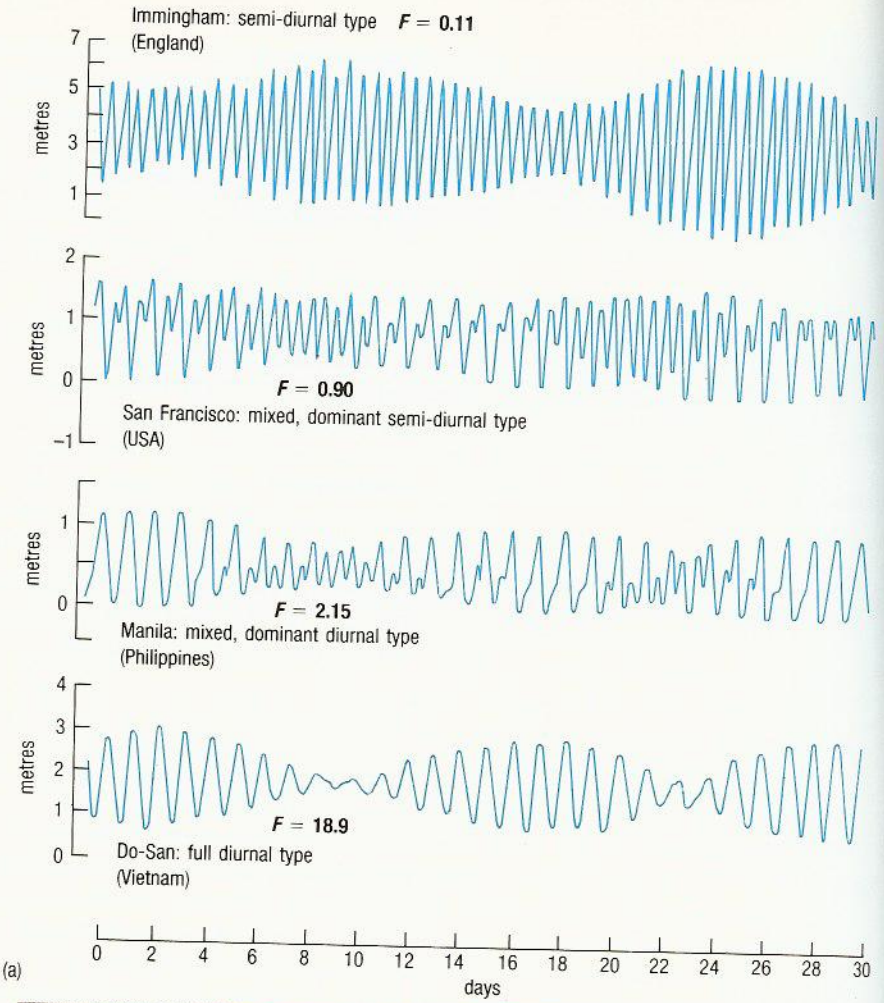
(a) From Figure 2.18, what are the main differences between tidal cycles characterized by high and low values of the ratio F?

(b) Would you expect the interval between spring tides to be 14.75 days (i.e. half of 29.5 days) at all times, and at all locations, irrespective of the other types of tidal fluctuation?

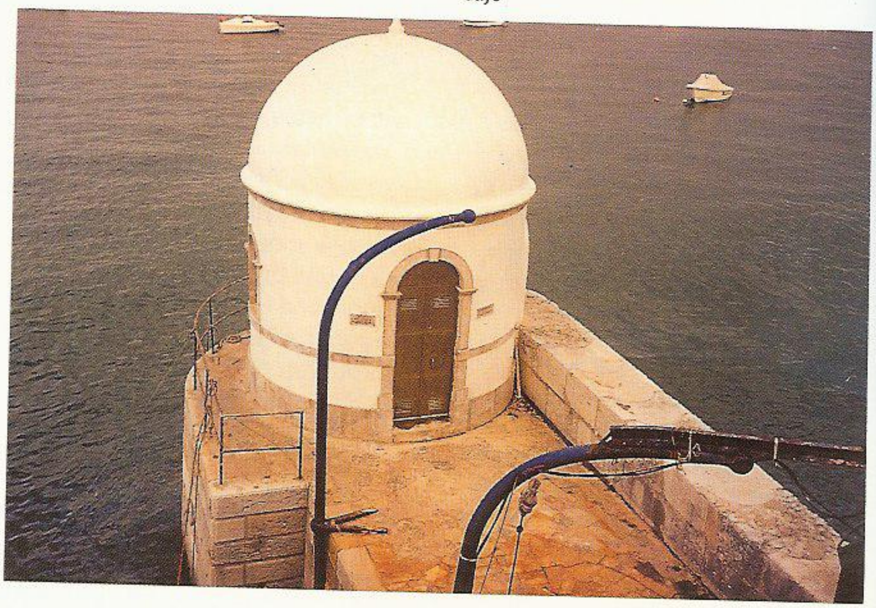
Figure 2.18 shows only a selection of the many possible types of tides that can occur. The actual tides at any particular location result principally from the combination of amplitude and phase of the diurnal and semi-diurnal constituents (Table 2.1) at that location. A high value of F (say above 3.0) implies a diurnal tidal cycle, i.e. only one high tide occurs daily, and fluctuations in tidal range are largely due to changes in the Moon's declination (Figure 2.8). Low values of F (say less than 0.25) imply a semi-diurnal tide, and the fluctuations in tidal range are mainly due to the relative positions of Sun and Moon, giving the spring–neap variation (Figure 2.12), and variations in lunar declination have only a relatively small effect.

Between these two extremes are the mixed tidal types, where daily inequalities are important, and there can be considerable variations in the amplitudes of, and time intervals between, successive high tides. The middle two tidal records in Figure 2.18(a) show diurnal inequalities where typical 'large tides' alternate with 'half-tides' (cf. Figure 2.17), and there is an additional contribution to the diurnal inequality resulting from the changing declination of the Moon, i.e. the change from tropic to equatorial tides and back again (Section 2.1.1). For example, the transition between tropic and equatorial tides can be seen at around days 6–9 and 19–22 in the record for San Francisco (Figure 2.18(a)), as lunar declination passes through zero. However, changes in the Moon's declination have less effect at higher latitudes, and diurnal inequalities are therefore not an obvious feature of tides around Britain, for example.

The configuration of an ocean basin determines its natural resonant period (Section 1.6.4), and along open ocean coasts the type of tide (Figure 2.18(a)) depends upon whether the adjacent ocean responds more readily to diurnal or semi-diurnal constituents of the tide-producing forces. In the Atlantic Ocean and most of the Indian Ocean, the response is mainly semi-diurnal, though the natural period of the Gulf of Mexico appears to be about 24 hours, and diurnal tides predominate there. In the Pacific Ocean, the diurnal response is more significant and tides are usually of the mixed type, though they are predominantly diurnal in northern and some western parts of the Pacific (cf. Figure 2.18(a)).



(a)



(b)

Figure 2.18 (a) Examples of different types of tidal curves, in England, the USA, the Philippines, and Vietnam. Note that the vertical scales are not the same in each record. For full explanation, see text.
(b) The tidal measuring station at Cascais harbour, Portugal, in use since the late 19th century, is one of the longest-serving stations in the European tidal gauge network. The recorder is at the top of a 'stilling' well which dampens the oscillations of swell and other wind waves entering the harbour.

It is worth mentioning here that, at any particular location, the highest and lowest spring tides will occur at the same times of day (~6 hr 25 min. apart). That is because the alternation of spring and neap tides is determined by the Sun (Figure 2.12) and the period of the S_2 constituent is 24 hr (Table 2.1). As you might expect, a similar relationship applies to neap tides. The feeding and reproductive behaviour of many marine animals, especially those living in nearshore and shallow shelf waters, is 'tuned' to tidal cycles, notably the 29.5-day lunar or synodic month (the spring–neap period, Section 2.2.1) – see also Section 2.4.1.

In shallow water, local effects can modify tidal constituents such as M_2, particularly by producing harmonics whose frequencies are simple multiples of the frequency of the constituent concerned. These harmonics result from frictional interactions between the sea-bed and the ebb and flow of the tide – especially in shallow waters. For example, the quarter-diurnal constituent M_4 (twice the frequency of M_2) and the one-sixth-diurnal constituent M_6 (three times the frequency of M_2) are generated in addition to the semi-diurnal constituents. In most locations, the effect of these two harmonics is insignificant compared with the principal constituents, but along the Dorset and Hampshire coasts of the English Channel each has a larger amplitude than usual. Moreover, the two harmonics are in phase, and their combined amplitude is significant when compared to that of M_2. (Just west of the Isle of Wight, M_2 is about 0.5 m, M_4 about 0.15 m, and M_6 about 0.2 m.) The additive effect of all three constituents causes the double high waters at Southampton and the double low waters at Portland. However, there is no truth in the popular myth that double high water at Southampton is caused by the tide flooding at different times around either end of the Isle of Wight.

The Mediterranean and other enclosed seas (e.g. Black Sea, Baltic Sea) have small tidal ranges of about 0.5 m or less, because they are connected to the ocean basins only by narrow straits. The tidal waves of the major amphidromic systems (Figure 2.15) cannot themselves freely propagate through these restricted openings. However, interaction between Atlantic tides and the shallow-water shelf region near Gibraltar for example, results in the generation of internal waves, which *do* propagate into the Mediterranean (Figure 1.23(b)) – and the internal waves seen in Figure 1.23(a) in the South China Sea may have a similar cause. By contrast, it is unlikely that similar packets of internal waves would occur where the Bosphorus connects to the Black Sea because the tidal range in the adjacent Mediterranean is negligible.

2.5 SUMMARY OF CHAPTER 2

1 Tides are long-period waves, generated by gravitational forces exerted by the Moon and Sun upon the oceans. They behave as shallow-water waves because of their very long wavelengths. Tidal currents are the horizontal water movements corresponding to the rise and fall (flood and ebb) of the tide.

2 A centrifugal force, directed away from the Moon, results from the Earth's (eccentric) rotation (period 27.3 days) around the Earth–Moon centre of mass, which is within the Earth. This centrifugal force is exactly balanced *in total* by the gravitational force exerted on the Earth by the Moon. However, gravitational force exceeds centrifugal force on the 'Moon-side' of Earth, resulting in tide-producing forces directed towards the Moon, whereas on the other side of the Earth centrifugal force exceeds gravitational force, resulting in tide-producing forces directed away from the Moon.

3 Tractive forces (horizontal components of tide-producing forces) are maximal on two small circles either side of the Earth, and produce two (theoretical) equilibrium tidal bulges – one directed towards the Moon, and the other directed away from it. As the Earth rotates with respect to the Moon (with a period of 24 hours 50 minutes), the equilibrium tidal bulges would need to travel in the opposite direction (relative to the surface of the rotating Earth) in order to maintain their positions relative to the Moon. The elliptical orbit of the Moon about the Earth causes variation in the tide-producing forces of up to 20% from the mean value.

4 With the Moon overhead at the Equator, the equilibrium tidal bulges would be in the same plane as the Equator, and at all points the two bulges would theoretically cause two equal high tides daily (equatorial tides). The Moon has a declination of up to 28.5° either side of the Equator, and when the plane of the tidal bulges is offset with respect to the Equator, there are two unequal, or tropic, tides daily. The declination varies over a 27.2-day cycle.

5 The Sun also produces tides which show inequalities related to the Sun's declination (up to 23.4° either side of the Equator), and vary in magnitude due to the elliptical orbit of the Earth around the Sun. The Sun's tide-producing force has about 46% of the strength of the Moon's. Solar tides combine with and interact with lunar tides. When Sun and Moon are in syzygy, the effect is additive, giving large-ranging spring tides; but when Sun and Moon are in quadrature, tidal ranges are small (neap tides). The full cycle (a lunar month), includes two neaps and two springs, and takes 29.5 days.

6 Tidal speed is limited to about 230 $m\,s^{-1}$ in the open oceans (less in shallower seas), and land masses constrain tidal flow. Water masses have inertia and experience friction with coasts and the sea-bed, so they do not respond instantaneously to tractive forces. The Coriolis force, and constraining effects of land masses, combine to impose amphidromic systems upon tides. High tidal crests circulate (as Kelvin waves) around amphidromic points which show no change in tidal level, i.e. tidal range increases with distance from an amphidromic point. Amphidromic systems tend to rotate in the opposite direction to the deflection caused by the Coriolis force.

7 The actual tide is made up of many constituents (partial tides), each corresponding to the period of a particular astronomical motion involving Earth, Sun or Moon. Partial tides can be determined from tidal measurements made over a long time at individual locations, and the results used to compute future tides. Actual tides are classified by the ratio (F) of the summed amplitudes of the two main diurnal constituents to the summed amplitudes of the two main semi-diurnal constituents.

8 Tidal rise and fall are produced by lateral water movements called tidal currents. Tidal current vectors typically display 'tidal ellipses' rather than simple to-and-fro motions.

9 Areas of low atmospheric pressure cause elevated sea-levels, whereas high pressure depresses sea-level. A strong wind can hold back a high tide or reinforce it. Storm surges are caused by large changes in atmospheric pressure and the associated strong winds. Positive storm surges may result in catastrophic flooding.

10 In estuaries, the tidal crest travels faster than the tidal trough because speed of propagation depends upon water depth; hence the low water to high water interval is shorter than that from high water to low water. Tidal bores develop where tides are constrained by narrowing estuaries and the wave-front is forced by the rising tide to travel faster than the depth-determined speed of a shallow-water wave. Where tidal ranges are large and the water can be trapped by dams, the resultant heads of water can be used for hydro-electric power generation.

Now try the following questions to consolidate your understanding of this Chapter.

QUESTION 2.11 Write an expression for the tide-producing force at point P on Figure 2.4(a), using the terms as defined for Equations 2.1, 2.2 and 2.3. It is not essential to try to simplify or approximate the expression.

QUESTION 2.12 Which of the following statements are true?

(a) 'In syzygy' has the same meaning as 'in opposition'.

(b) Neap tides would be experienced during an eclipse of the Sun.

(c) Spring tides do not occur in the autumn.

(d) The lowest sea-levels of the spring–neap cycle occur at low tide while the Moon is in quadrature.

QUESTION 2.13 Briefly summarize the factors accounting for differences between the equilibrium tides and the observed tides.

QUESTION 2.14 How will each of the following influence the tidal range at Immingham (Figure 2.18(a)):

(a) The Earth's progress from perihelion to aphelion?

(b) The occurrence of a tropic tide?

(c) A 30 millibar rise in atmospheric pressure?

CHAPTER 2

Question 2.1 (a) The magnitudes of the gravitational and centrifugal force and hence of the resultant tide-producing force would be the same as at D and J on Figure 2.3. The tide-producing force would be directed into the Earth (into the plane of the page as you look at Figure 2.3).

(b) This is the only point where the gravitational force exerted by the Moon on the Earth is exactly equal to, and acting in exactly the opposite direction from, the centrifugal force. The resultant tide-producing force at that point is zero.

Question 2.2 (a) The waves would be required to travel 40 000 km in 24 hr 50 min, i.e. in 24.83 hours. That is equivalent to $1611\,\text{km}\,\text{hr}^{-1}$, or $448\,\text{m}\,\text{s}^{-1}$.

(b) Using Equation 1.4, if $c = \sqrt{gd}$, then $d = c^2/g$.

So, depth required = $448^2/9.8$ = 20 480 m, i.e the ocean would have to be more than 20 km deep.

Question 2.3 (a) Nil. About seven days (i.e. one-quarter of 27.3 days) after the scenario shown on Figure 2.9, the Moon will be overhead at the Equator (Figure 2.5), which occurs at positions 2 and 4 in Figure 2.8. There will then be no diurnal inequality in the lunar tide anywhere on the globe.

(b) Diurnal inequality will again be at a maximum, and the tidal bulges will be as in Figure 2.9 once more, although the Moon will be on the opposite side of the Earth (corresponding to position 3 on Figure 2.8).

Question 2.4 The tidal contribution from the Sun will be greatest around 3 January, when the Earth is at perihelion (which happens to be quite close to the time of the winter solstice, 21 December).

Question 2.5 (a) 14.75 days (half of 29.5).

(b) Neap tides. Spring tides coincide with syzygy, so 14.75 days after that there will be another spring tide, and 7.4 days more will bring the cycle to neap tide (14.75 + 7.4 = 22.15).

(c) 3–4 days, i.e. half-way between the spring tide associated with the new Moon and the neap tide which will occur 7.4 days after the new Moon.

(d) In this simplest case, Figure 2.12(a) corresponds to a solar eclipse (Moon directly between Sun and Earth) and Figure 2.12(c) to a lunar eclipse (Earth's shadow on Moon).

Question 2.6 (a) (1) *The Wash:* Just after low tide (i.e. nearly 6 hours to go to high tide).

(2) *Firth of Forth:* About $1\frac{1}{2}$ lunar hours to go to high tide (i.e. about $4\frac{1}{2}$ hours after low tide).

If you had difficulty with (a) and (b), note that if high tide is at '0' on Figure 2.14, it has not yet reached the Firth of Forth, and that area is expecting a high tide in two hours' time. Similarly, the high tide will take five-and-a-half hours to reach the Wash.

(b) The tidal range of the Wash (over 6 m) exceeds that of the Firth of Forth (more than 4 m, but less than 5 m).

Question 2.7 (a) The amphidromic systems in the South Atlantic, mid-Pacific and round Madagascar are all in the Southern Hemisphere and rotate anticlockwise rather than clockwise as the theory would suggest; whereas the system in the North Pacific is in the Northern Hemisphere and rotates clockwise, not anticlockwise. It may be worth noting that the mid-Pacific and North Pacific amphidromic points are also quite close to islands (the Tuamoto and Hawaiian groups respectively).

(b) The amphidrome centred in the north-west Indian Ocean would seem to be sufficiently enclosed to fulfil the criteria illustrated in Figure 2.16. Being in the Northern Hemisphere, it should rotate anticlockwise, but it is in fact a clockwise system. The reason for this may be that there are in fact two systems in the northern Indian Ocean, both centred nearly on the Equator where the Coriolis force is zero.

Question 2.8 (a) As you might expect, a high value of F corresponds to a dominant diurnal tidal component (average period of 24 hr 50 min). A low value of F indicates dominance of a semi-diurnal component.

(b) Yes. Syzygy occurs every 14.75 days, and as noted in the text following Question 2.5 it affects tides all over the world simultaneously.

Question 2.9 (a) Using Equation 1.18, with $l = 270$ km, and $d = 60$ m, we get:

resonant period of basin = $4l/\sqrt{gd} = 4 \times 270 \times 10^3/\sqrt{9.8 \times 60} = 44\,538$ s = 12.37 hours.

This is very close to the semi-diurnal period of 12.42 hours, i.e. the resonant period of the basin is the same as that of the semi-diurnal tides. The reason for using Equation 1.18 is that the Bay of Fundy is a basin open at one end. Equation 1.17 is for standing waves in closed basins.

(b) From the calculation in (a) and Figure 1.20(c), the wavelength of the standing wave would be 4×270 km, which works out to 1080 km, i.e. the wavelength is of the order of 10^3 km.

(c) We would expect the tidal range to be small near the entrance to the Bay of Fundy, because Figure 1.20(c) suggests this is where the node of the standing wave should be – and by definition, *vertical* displacements of the water surface there should be minimal, cf. Figure 1.20(d).

Question 2.10 If a 10 m column of water (= 1000 cm) corresponds to one atmosphere of pressure (1000 mbar), then 1 cm of water corresponds to 1 mbar. A reduction in atmospheric pressure of 50 mbar would therefore result in a sea-level *rise* of 50 cm, i.e. 0.5 m.

Question 2.11 The tide-producing force at P (TPF_P) = the Moon's gravitational attraction (F_{gP}) at P minus the centrifugal force at P.

$$F_{gP} = \frac{GM_1M_2}{(R - a\cos\psi)^2} \qquad \text{(Eqn 2.3)}$$

and the centrifugal force at P is the same as at all other points on Earth, and therefore equal to F_g at the Earth's centre, i.e. GM_1M_2/R^2 (from Eqn 2.1). So by analogy with the reasoning leading to Equation 2.2:

$$TPF_P = \frac{GM_1M_2}{(R - a\cos\psi)^2} - \frac{GM_1M_2}{R^2}$$

Do not worry if you could not manage the subsequent algebra, but the expression simplifies to:

$$TPF_P = GM_1M_2 \frac{a\cos\psi(2R - a\cos\psi)}{R^2(R - a\cos\psi)}$$

This can be simplified to the approximate relationship:

$$TPF_P \approx \frac{GM_1M_2 2a\cos\psi}{R^3}$$

Question 2.12 None of the statements is true.

(a) False. The term 'syzygy' includes *both* conjunction and opposition. If the Moon is in opposition, it is also in syzygy. However, if the Moon is in syzygy, it is not necessarily in opposition (i.e. it might be in conjunction).

(b) False. Spring tides would occur (see Figure 2.12(a)).

(c) False. Spring tides occur every 14.75 days throughout the year.

(d) False. The lowest sea-levels occur at low *spring* tides. When the Moon is in quadrature, neap tides occur, and these have a smaller range than spring tides.

Question 2.13 The actual observed tides are constrained by the shallow depth of the oceans, and also by inertia, friction and shape of the ocean basins. So there is a time-lag between the application of the tide-generating forces and the oceans' responses. In addition, tidal currents are subject to the Coriolis force.

Question 2.14 (a) There would be a decrease in the tidal range. The difference between perihelion and aphelion in terms of Earth–Sun distance is only about 4% (Section 2.2), and the Sun has slightly less than half the tide-producing influence of the Moon. The decrease in tidal range as the Earth–Sun distance increased would therefore be small (say a centimetre or so).

(b) As noted in Section 2.4, changes in the Moon's declination, which cause equatorial and tropic tides (Section 2.1.1), have less effect at higher latitudes. As Figure 2.18(a) shows, there is little diurnal inequality in the tidal range at Immingham (F-value = 0.1), although close inspection of the tidal curve does reveal very slight diurnal inequalities around days 6–8 and 21–23.

(c) No effect on tidal *range*, but by analogy to the answer to Question 2.10 a depression of the sea-surface of about 30 cm would result, whatever the state of the tide at the time.