CHAPTER 2 TIDES

'... being governed by the watery Moon ...'

Richard III, Act II, Scene II.

The longest oceanic waves are those associated with the tides, and are characterized by the rhythmic rise and fall of sea-level over a period of half a day or a day (Figure 1.2). The rise and fall result from horizontal movements of water (tidal currents) in the tidal wave. The rising tide is usually referred to as the flow (or **flood**), whereas the falling tide is called the **ebb**. The tides are commonly regarded as a coastal phenomenon, and those who see tidal fluctuations only on beaches and in estuaries tend to think (and speak) of the tide as 'coming in' and 'going out'. However, it is important to realize that the ebb and flow of the tide at the coast is a manifestation of the general rise and fall in sea-level caused by a long-wavelength wave motion that affects the oceans as well as shallow coastal waters. Nonetheless, because of their long period and wavelength (Figure 1.2), tidal waves behave as *shallow-water waves*. Do bear in mind also, from Section 1.6.3, that the destructive waves generated by earthquakes are not 'tidal waves' as so often reported in the press – they are *tsunamis*, which also behave as shallow-water waves because of their long wavelength.

From the earliest times, it has been realized that there is some connection between the tides and the Moon. High tides are highest and low tides are lowest when the Moon is full or new, and the times of high tide at any given location can be approximately (but not exactly) related to the position of the Moon in the sky; and, as we shall see, the Sun also influences the tides.

Before discussing these relationships, we shall first describe some principal features of tidal wave motions. Figure 2.1 is a tidal record, showing regular vertical movements of the water surface relative to a mean level, over a period of about a month.

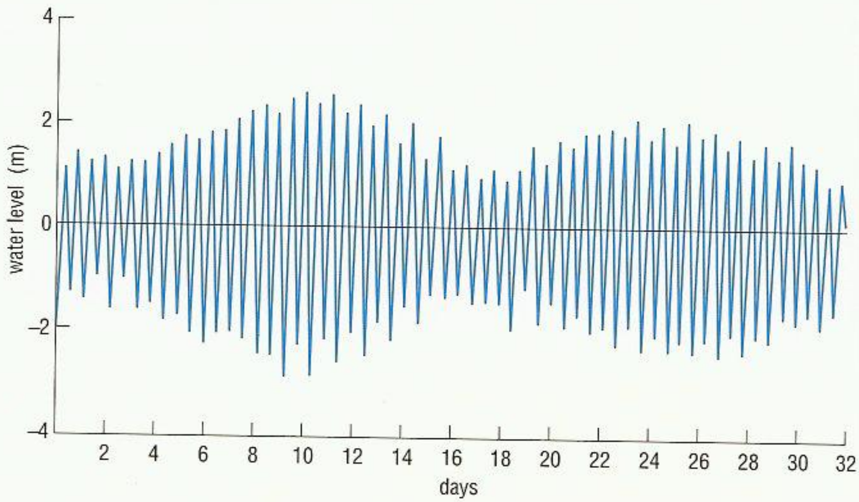

Figure 2.1 A typical 30-day tidal record showing oscillations in water level with a period of about 12.5 hours, at a station in the Tay estuary, Scotland.

If you compare Figures 2.1 and 1.5, you will see two important differences between wave motions resulting from the tides and those associated with wind-generated waves. These are:

1 The period of the oscillations of wind-generated waves (Figure 1.5) is typically in the order of seconds to a few tens of seconds, and both period and amplitude of the oscillations can be quite irregular. In contrast, Figure 2.1 shows the period of the tides to be about 12.5 hours, i.e. high and low tides occur twice a day, and both period and amplitude vary in a systematic way. (Figure 2.1 illustrates a semi-diurnal tide; we shall consider the different types of tide later.)

2 Although the amplitude (and height) of tidal and wind-generated wave motions is of the same order in both Figures 1.5 and 2.1, we have seen that the heights of wind waves can range from virtually zero to 30 m or more (Section 1.6.2). By contrast, in most places the **tidal range** is typically of the order of a few metres, and tidal ranges of more than about 10 m are known only at a few locations. Tidal range nearly always varies within the same limits at any particular location (Figure 2.1), and because the cause of tidal wave motion is both continuous and regular, so that the periodicities that result are pre-determined and fixed (as you will see shortly), tidal range can be very reliably predicted. Wind-generated waves, on the other hand, are much less predictable, because of the inherent variability of the winds. Tidal waves are what are known as 'forced waves' because they are generated by regular (periodic) external forces, and therefore do not behave exactly like the gravity waves considered in Chapter 1. For practical purposes, however, they can be treated as gravity waves, especially in the deep oceans.

In addition to the ~12-hour period of oscillations in Figure 2.1, can you discern another periodic variation?

A 7–8-day periodicity can also be seen: around days 9 to 11 and 23 to 26, the tidal range is more than twice what it is around days 0 to 2 and 16 to 18. This 7–8-day alternation of high and low tidal range (*spring* and *neap* tides, respectively) can also be predicted with great accuracy and characterizes tides all over the world (see Section 2.2.1).

What are the ranges of the spring and neap tides on Figure 2.1?

Spring tides have an amplitude of nearly 3 m (i.e. above and below the mean water level), so the spring tidal *range* is close to 6 m. In contrast, the neap tides have a range of little more than 2 m.

Where there is urban or industrial development in coastal areas, it is common for high and low tidal levels to be quite rigorously identified, because along gently sloping shorelines a tidal range of even a couple of metres results in substantial areas of ground being alternately covered and exposed by the flooding and ebbing tides. In coastal areas, maps and plans commonly indicate Mean High Water and Mean Low Water, as well as the Mean Tide Level. The Mean Tide Level is often used as a datum or baseline for topographic survey work, i.e. it is the baseline for all measurements of elevation and depth on maps and charts. For example, in Britain, this baseline (known as the Ordnance Datum) is the Mean Tide Level at a specific location at Newlyn in Cornwall.

This discussion of tidal levels raises an important general point about people's perception of the tides. As mentioned earlier, those who see tidal fluctuations only on beaches or in estuaries tend to perceive the tide as 'coming in' and 'going out'. In fact, the sea advances over and retreats from the land *only* because the water level is rising and falling with the passage of tidal waves like those illustrated in Figure 2.1.

So much for some basic descriptions of the tides. We must now consider the forces that cause them. The relative motions of the Earth, Sun and Moon are complicated, and so their influence on tidal events results in an equally complex pattern. Nevertheless, as we have just seen, the actual motions of the tides are quite regular, and the magnitudes of the tide-generating forces can be precisely formulated. Although the response of the oceans to these forces is modified by topography and by the transient effects of weather patterns, it is possible to make reliable predictions of the tides for centuries ahead (and indeed to relate specific historical events to tidal states many centuries in the past).

2.1 TIDE-PRODUCING FORCES – THE EARTH–MOON SYSTEM

The Earth and the Moon behave as a single system, rotating about a common centre of mass, with a period of 27.3 days. The orbits are in fact elliptical, but to simplify matters we will treat them as circular for the time being. The Earth rotates eccentrically about the common centre of mass (centre of gravity), which is within the Earth and lies about 4700 km from its centre. Figure 2.2 illustrates the motions that result. The principal consequence of the eccentric motion about the Earth–Moon centre of mass is this: All points on and within the Earth must also rotate about the common centre of mass and so they must all follow the same elliptical path. So each point must have the same angular velocity ($2\pi/27.3$ days), and hence will experience the same centrifugal force (which is proportional to acceleration towards the centre, i.e. to the product of the radius and the square of the angular velocity).

The eccentric motion described above *has nothing whatsoever to do with the Earth's rotation (spin) upon its own axis, and should not be confused with it* (we have shown the Earth's rotation axis on Figure 2.2 for the situation where the Moon is directly above the Equator, which happens only twice every 27.3 days – see Section 2.1.1 and Figure 2.8). Nor should the centrifugal force resulting from the eccentric motion (which is equal at all points on Earth) be confused with the centrifugal force caused by the Earth's spin (which increases with distance from the rotation axis).

If you find these concepts difficult, the following simple analogy may help. Imagine you are whirling a small bunch of keys on a short length (say 25 cm) of chain. The keys represent the Moon, and your hand represents the Earth. You are rotating your hand eccentrically (but unlike the Earth it is not spinning as well), and all points on and within your hand are experiencing the same angular velocity and the same centrifugal force. Provided your bunch of keys is not too large, the centre of mass of the 'hand-and-key' system lies within your hand.

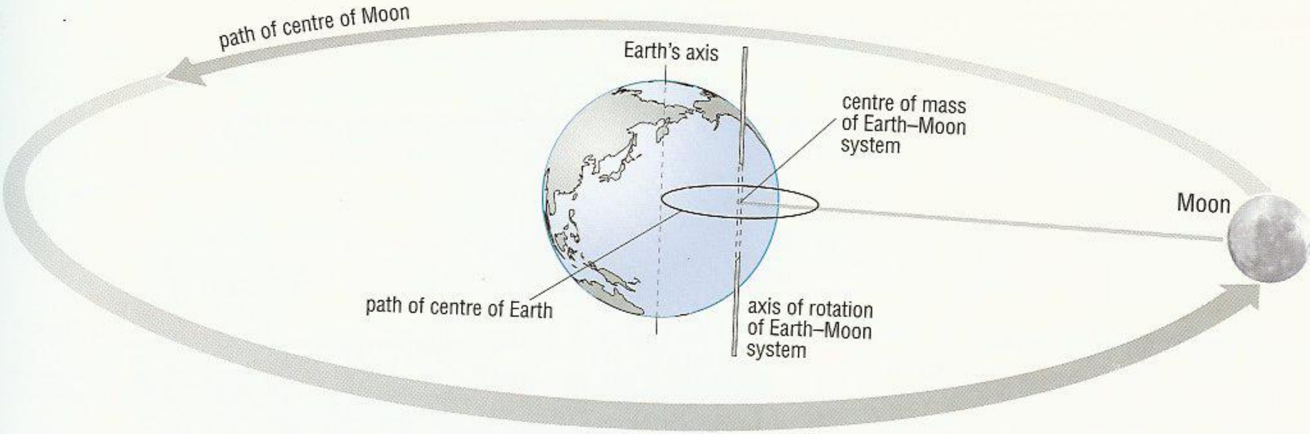

Figure 2.2 Rotation of the Earth–Moon system (not to scale). The Moon orbits the Earth about their common centre of mass (located within the Earth) once every 27.3 days. The centre of the Earth also rotates about this centre of mass once every 27.3 days, describing a very much smaller orbit (fine black line), as do all other points on and within the Earth. Note that the orbits are shown as circular for simplicity, whereas in fact they are elliptical (see later text); note also that the Earth's own central rotation axis is shown here as perpendicular to the plane of the Moon's orbit, which happens twice every 27.3 days – see Figure 2.8.

In the text which follows, you do not need to understand the details of the explanation related to Figures 2.3 and 2.4. However, you do need to be aware of the relationship embodied in Equation 2.2 on p. 55, i.e. that tide-producing forces are inversely proportional to the *cube* of the Earth–Moon distance, and that the tide-producing forces are greatest along the small circles shown in Figure 2.4(a).

The total centrifugal force acting on the Earth–Moon system exactly balances the forces of gravitational attraction between the two bodies, so the system is in equilibrium, i.e. we should neither lose the Moon, nor collide with it, in the near future. The centrifugal forces are directed parallel to a line joining the centres of the Earth and the Moon (see red arrows on Figure 2.3, overleaf). Now consider the gravitational force exerted by the Moon on the Earth. Its magnitude will not be the same at all points on the Earth's surface, because they are not at the same distance from the Moon. Points nearest the Moon will experience a greater gravitational pull from the Moon than those on the opposite side of the Earth. Moreover, the direction of the Moon's gravitational pull at all points will be directed towards its centre (see blue arrows on Figure 2.3), so it will not be exactly parallel to the direction of the centrifugal forces, except along the line joining the centres of the Earth and Moon.

The resultant (i.e. the composite effect) of the two forces is known as the **tide-producing force**. Depending upon its position on the Earth's surface with respect to the Moon, this force is directed into, parallel to, or away from, the Earth's surface. Its direction and relative strength (not strictly to scale) is shown by thick purple arrows on Figure 2.3.

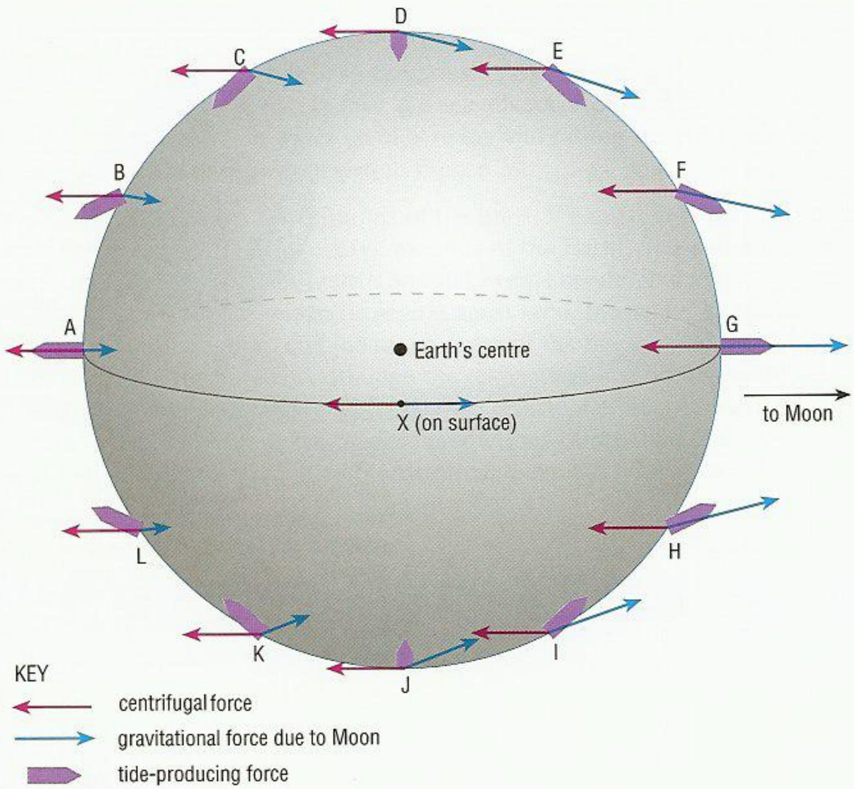

Figure 2.3 The derivation of the tide-producing forces (not to scale), for a hypothetical water-covered Earth. The centrifugal force has exactly the same magnitude and direction at all points, whereas the gravitational force exerted by the Moon on the Earth varies in both magnitude (inversely with the square of the distance from the Moon) and direction (directed towards the Moon's centre, but shown with the angles exaggerated for clarity). The tide-producing force at any point (thick purple arrows) is the *resultant* of the gravitational and centrifugal force at that point, and varies inversely with the *cube* of the distance from the Moon (see text).

QUESTION 2.1 What would be the direction and approximate magnitude (within the context of Figure 2.3) of the tide-producing forces at:

(a) a point on the Earth's surface represented by point X on Figure 2.3?

(b) the Earth's centre?

The gravitational force (F_g) between two bodies is given by:

$$F_g = \frac{GM_1M_2}{R^2} \tag{2.1}$$

where M_1 and M_2 are the masses of the two bodies, R is the distance between their centres, and G is the universal gravitational constant (whose value is $6.672 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$).

However, we need to reconcile Equation 2.1 with the statement in the caption to Figure 2.3 that the magnitude of the tide-producing force exerted by the Moon on the Earth varies inversely with the *cube* of the distance. Consider the point marked G on Figure 2.3. The gravitational attraction of the Moon at G (F_{gG}) is greater there than that at the Earth's centre, because G is nearer to the Moon by the distance of the Earth's radius (a). The gravitational force exerted by the Moon at the Earth's centre is exactly equal and opposite to the centrifugal force there, so the tide-producing force at the centre of the Earth is zero. Now as the centrifugal force is equal at all points on Earth, and at the Earth's centre is equal to the gravitational force exerted there by the Moon, it follows that we can substitute the expression on the right-hand side of Equation 2.1 (i.e. GM_1M_2/R^2) for the centrifugal force.

The tide-producing force at point G (TPF_G) is given by the force due to gravitational attraction of the Moon at G (F_{gG}) minus the centrifugal force at G, i.e.

$$TPF_G = \frac{GM_1M_2}{(R-a)^2} - \frac{GM_1M_2}{R^2} \text{s}$$

which simplifies to:

$$TPF_G = \frac{GM_1M_2a\,(2R-a)}{R^2(R-a)}$$

Now a is very small compared to R, so $(2R - a)$ can be approximated to $2R$, and $(R - a)^2$ to R^2, giving the relationship:

$$TPF_G \approx \frac{GM_1M_2\,2a}{R^3} \tag{2.2}$$

In other words, the tide-producing force is proportional to $1/R^3$.

Before reading on, have another look at Figure 2.3, and consider at which of the lettered points on that Figure the local tide-producing force would have most effect in generating tides.

You may have considered point G as your answer. Certainly, G is nearest to the Moon, and hence is one of the two points where the difference between the centrifugal force and the gravitational force exerted by the Moon is greatest. However, at point G all the resultant tide-producing force is acting vertically against the pull of the Earth's own gravity, which happens to be about 9×10^6 greater than the tide-producing force. Hence the local effect of the tide-producing forces at point G is negligible. Similar arguments apply at point A, except that the gravitational attraction of the Moon at point A (F_{gA}) is *less* than the centrifugal force, and consequently the tide-producing force at A is equal in magnitude to that at G, but directed *away* from the Moon (Figure 2.3).

The points we need to identify are those where the horizontal component of the tide-producing force, i.e. the **tractive force**, is at a maximum. Such points do not lie directly on a line joining the centres of the Earth and Moon, and so Equation 2.2 becomes slightly more complex. For example, at point P on Figure 2.4(a) the gravitational attraction (F_{gP}) would be, to a first approximation:

$$F_{gP} = \frac{GM_1M_2}{(R - a\cos\psi)^2} \tag{2.3}$$

The length $a\cos\psi$ is marked on Figure 2.4(a) (ψ is the Greek letter 'psi').

Equations such as 2.3 can be used to show that the tractive force is greatest at points along the small circles defined in Figure 2.4(a), *which have nothing to do with latitude or longitude.*

It is the tractive force that causes the water to move, because this horizontal component (by definition parallel to – i.e. tangential to – the Earth's surface at the location concerned) is unopposed by any other lateral force (apart from friction at the sea-bed, which is negligible in this context). The gravitational force due to the Earth is much greater than the tractive force but acts at right angles to it and so has no effect. The longest arrows on Figure 2.4(b) show where on the Earth the tractive forces are at a maximum when the Moon is over the Equator.

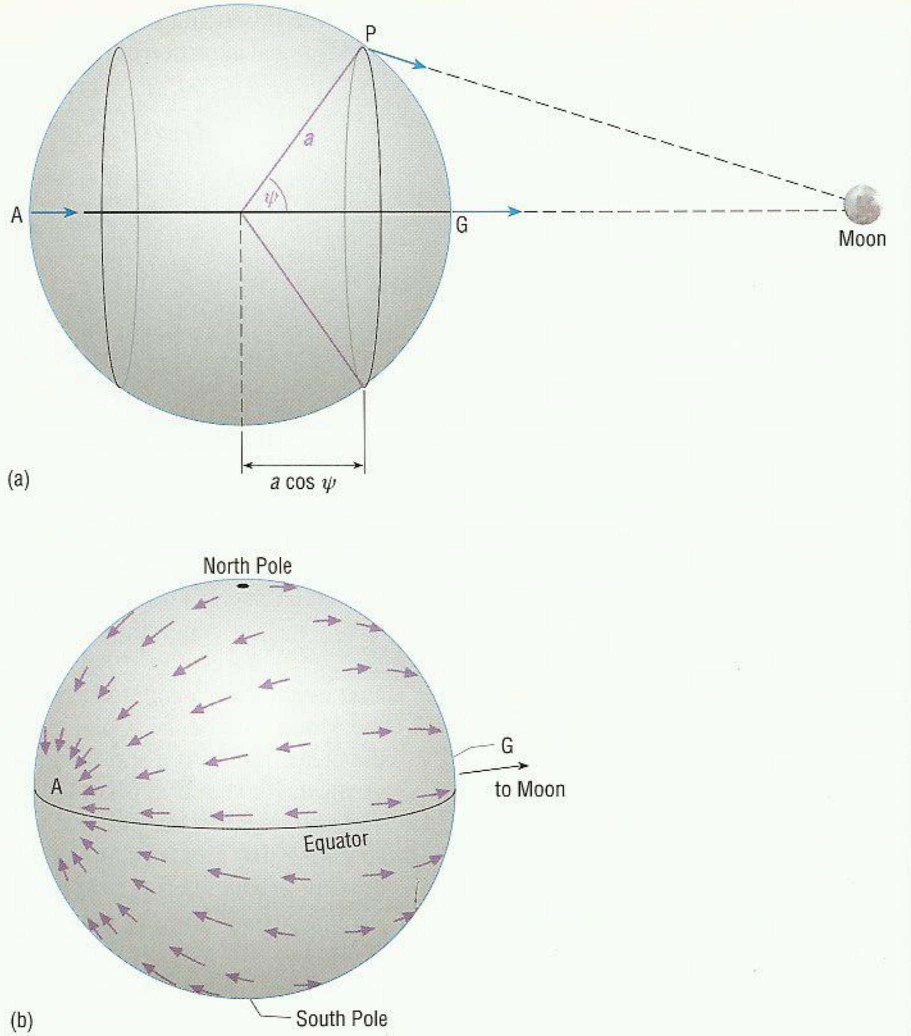

Figure 2.4 (a) The effect of the gravitational force of the Moon at three positions on the Earth. The gravitational force is greatest at G (nearest the Moon) and least at A (furthest from the Moon). At P the gravitational force is less than at G, and can be calculated from Equation 2.3. The tide-producing forces are smallest at A and G, but greatest at P, and all other points on the two small circles. The value for the angle ψ for these circles is 54° 41´. *The circles have nothing to do with latitude and longitude.* For explanation, see text.
(b) The relative magnitudes of the tractive forces (i.e. of the horizontal components of the tide-producing forces, shown as purple arrows on Figure 2.3) at various points on the Earth's surface. The Moon is assumed to be directly over the Equator (i.e. at zero declination, see Section 2.1.1). Points A and G correspond to those on (a) and in Figure 2.3.

In this simplified case, the tractive forces would result in movement of water towards points A and G on Figure 2.4(b). In other words, an equilibrium state would be reached (called the **equilibrium tide**), producing an ellipsoid with its two bulges directed towards and away from the Moon. So, paradoxically, although the tide-producing forces are minimal at A and G, those are the points towards which the water would tend to go. Figure 2.5 shows how such an equilibrium tidal ellipsoid would look in the simplified case we have been considering, i.e. a completely water-covered Earth with the Moon directly above the Equator and the distribution of tractive forces as in Figure 2.4(b).

If you found Figures 2.3 and 2.4 and related text and equations difficult to follow, here is a shorter explanation of why there are two equilibrium tidal bulges (Figure 2.5). The centrifugal force acts in the same direction all over the Earth, i.e. *away from* the Moon (Figure 2.3). Moreover, on the side of the Earth away from the Moon, the gravitational attraction due to the Moon is less than it is on the side of the Earth facing the Moon. The resultant tide-producing force thus acts *away from* the Moon at points such as A on Figure 2.3. That is why there is a tidal bulge away from the Moon as well as a bulge towards it (Figure 2.5). The mathematics of the relationship is such that theoretically the corresponding tide-producing forces on either side of the Earth are equal and opposite.

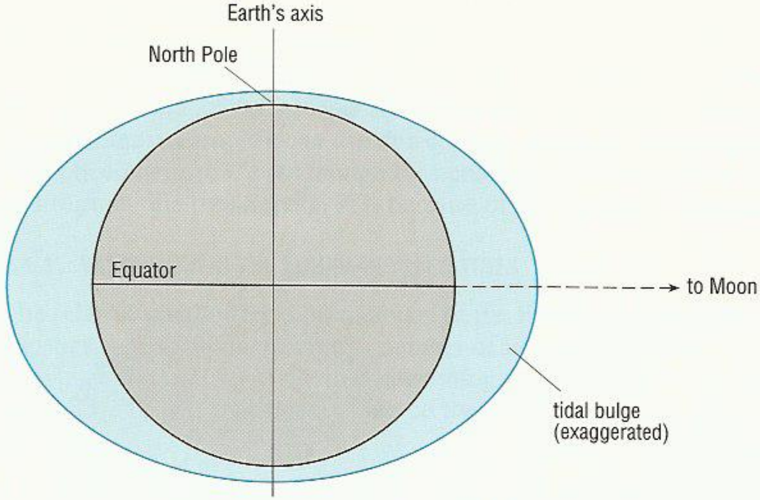

Figure 2.5 The equilibrium tidal ellipsoid (not to scale) as it would appear on a water-covered Earth with the Moon directly above the Equator.

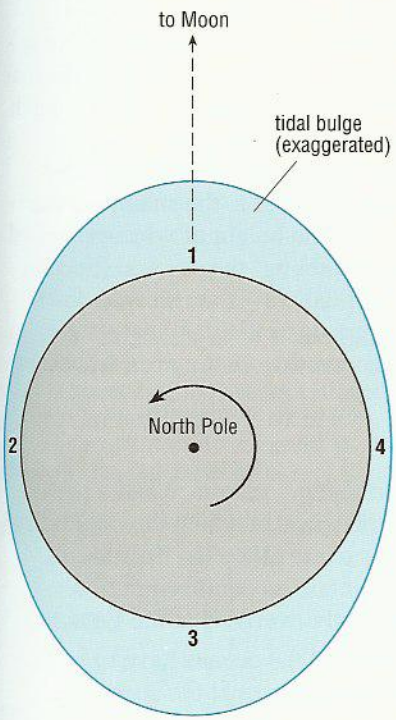

Figure 2.6 Rotation of the Earth within the equilibrium tidal bulge (seen from above the North Pole and not to scale), showing how a point on the Earth's surface would experience two high tides (1 and 3) and two low tides (2 and 4) during each complete rotation of the Earth about its axis.

In practice, the equilibrium ellipsoid does not develop, partly because the Earth is not of course entirely water-covered, but chiefly because the Earth rotates about its own axis. If the two bulges were to maintain their positions relative to the Moon, they would have to travel around the world at the same rate (but in the opposite direction) as the Earth rotates about its axis. Any point on the Earth's surface would thus encounter two high and two low tides during each complete rotation of the Earth (i.e. each day), as illustrated in Figure 2.6.

In fact, Figure 2.6 is an oversimplification. Can you see why (apart from the idealized tidal bulges)?

Figure 2.6 shows both Moon and tidal bulges remaining stationary during a complete rotation of the Earth. That cannot be the case, for the Moon continues to travel in its orbit as the Earth rotates. Because the Moon revolves about the Earth–Moon centre of mass once every 27.3 days, in the same direction as the Earth rotates upon its own axis (which is once every 24 hours), the period of the Earth's rotation with respect to the Moon is 24 hours and 50 minutes. This is the **lunar day**.

What effect would this have upon the interval between successive high tides and successive low tides, in Figure 2.6?

The interval between successive high (and low) tides would be about 12 hours 25 minutes – and the interval between high and low tide would be close to 6 hours $12\frac{1}{2}$ minutes. This is the reason why the times of high tides at many locations are almost an hour later each successive day (Figure 2.7, overleaf).

The equilibrium tidal concept also brings out another very important aspect of tidal wave motions.

Looking at Figures 2.5 and 2.6, would you say that tidal waves are more likely to travel as deep- or as shallow-water waves?

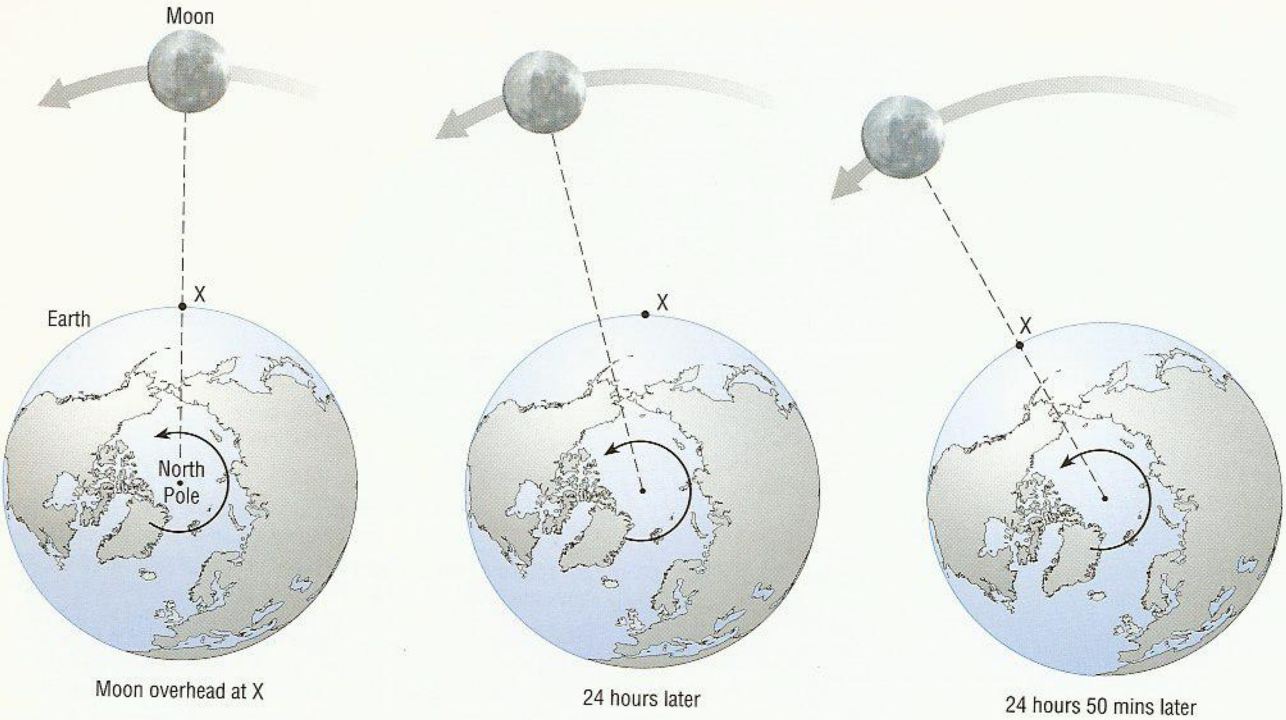

Figure 2.7 The relationship between a solar day of 24 hours and a lunar day of 24 hours and 50 minutes as seen from above the Earth's North Pole. Point X on the Earth's surface when the Moon is directly overhead comes back to its starting position 24 hours later. Meanwhile, the Moon has moved on in its orbit, so that point X has to rotate further (another 50 minutes' worth) before it is once more directly beneath the Moon. (Diagram not to scale.)

There are two 'peaks' (high tide, 1 and 3 in Figure 2.6) and two 'troughs' (low tide, 2 and 4 in Figure 2.6) for one Earth circumference, which is about 40 000 km. So the wavelength of the bulges in Figures 2.5 and 2.6 is of the order of half the Earth's circumference (~20 000 km). Even in the real oceans, tidal wavelengths are many thousands of km, and the average depth of the ocean basins is less than 4 km, i.e. much less than 1/20 of the wavelength (Section 1.2.3). So tidal waves must travel as shallow-water waves, and their speed is governed by Equation 1.4, i.e. the shallower the water the slower they travel. Moreover, just as the height of wind-generated waves increases as they are slowed down on 'feeling' the sea-bed (Section 1.5), so also does tidal range increase as the tidal waves are slowed down over the continental shelf. Tidal ranges are greater and tidal currents are therefore faster in shallow seas and along coasts than in the open oceans (cf. Figures 2.14 and 2.15).

QUESTION 2.2

(a) Using a value of 40 000 km for the Earth's circumference and a period of 24 hr 50 min. (the lunar day), calculate the speed at which the tidal bulges would have to move relative to the Earth's surface along the Equator, in order to 'keep up' with the Moon and so maintain an equilibrium tide. (Assume for simplicity that the Moon is directly overhead at the Equator.)

(b) According to Equation 1.4, how deep would the oceans have to be to allow the tidal bulges to travel as shallow-water waves at the speed you calculated in part (a)?

Your answer to Question 2.2 shows that in practice an equilibrium tide cannot occur at low latitudes on Earth – though it could in principle do so at high latitudes, where distances round the Earth are much less.

The concept of the equilibrium tide was developed by Newton in the seventeenth century, and we have seen that it demonstrates the fundamental periodicity of the tides on a semi-diurnal basis of 12 hours and 25 minutes (Figures 2.6 and 2.7), also that tidal waves must travel as shallow-water waves in the oceans. We can use this concept to explore other aspects of tidal phenomena too, even though the actual tides cannot behave like the equilibrium tide (see Section 2.3) because of the existence of continents.

2.1.1 VARIATIONS IN THE LUNAR-INDUCED TIDES

The relative positions and orientations of the Earth and Moon are not constant, but vary according to a number of interacting cycles. As far as a simple understanding of the tide-generating mechanism is concerned, only two cycles have a significant effect on the lunar tides.

1 The Moon's declination

The Moon's orbit is not in the plane of the Earth's Equator, but is inclined to it (Figure 2.8). This means that a line joining the centre of the Earth to that of the Moon makes an angle ranging from zero up to 28.5° on either side of the equatorial plane (see later text). This angle is the **declination** of the Moon. The result is that, to an observer on Earth, successive paths of the Moon across the sky appear to rise and fall over the 27.3-day period of rotation of the Moon about the Earth (strictly, about the centre of mass of the Earth–Moon system, Figure 2.2), in a similar way to the seasonal variation of the Sun's apparent daily path across the sky over the course of a year (i.e. lower in the sky in winter, higher in the summer, see Figure 2.11).

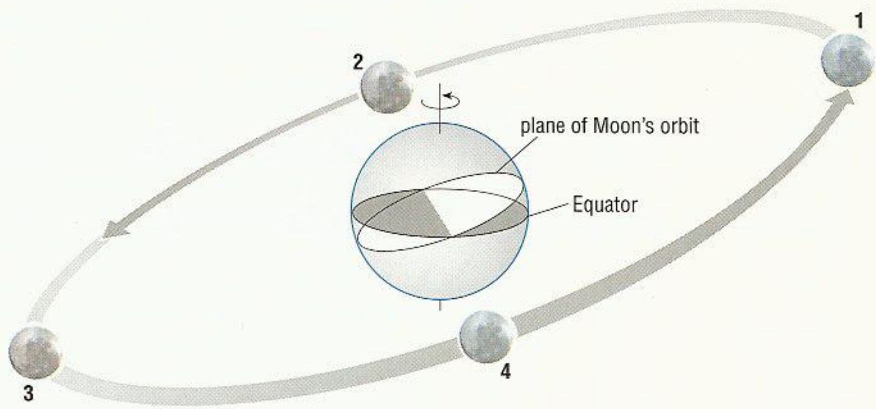

Figure 2.8 Declination of the Moon results from the plane of the Moon's orbit being at an angle to that of the Earth's Equator (shaded). For numbers, see text. Diagram is not to scale, and the Moon's orbit is shown as circular for simplicity (see item 2 overleaf).

At which of the numbered positions of the Moon in its orbit in Figure 2.8 is the declination at a maximum and at which is it zero? What is the time interval between the successive numbered positions on Figure 2.8?

Declination is maximum at positions 1 and 3, and zero at positions 2 and 4 when the Moon is overhead at the Equator. The interval between successive numbered positions in Figure 2.8 is close to seven days (27.3/4). Since the maximum lunar declination is 28.5°, the Moon can never be seen directly overhead poleward of latitude 28.5° N or 28.5° S. So, for example, in southern Britain at about 50° N, the Moon (like the Sun) is always seen in the southern sky. Conversely, in Tasmania for example, at about 40° S, the Moon (like the Sun) is always seen in the northern sky.

When the Moon is at any angle of declination other than zero, the plane of the two tidal bulges will be offset with respect to the Equator, and their effects at a given latitude will be unequal, particularly at mid-latitudes. Hence the heights reached by the semi-diurnal (i.e. twice daily) high tides will show diurnal (i.e. daily) inequalities (Figure 2.9); these will be greatest when the Moon is at maximum declination.

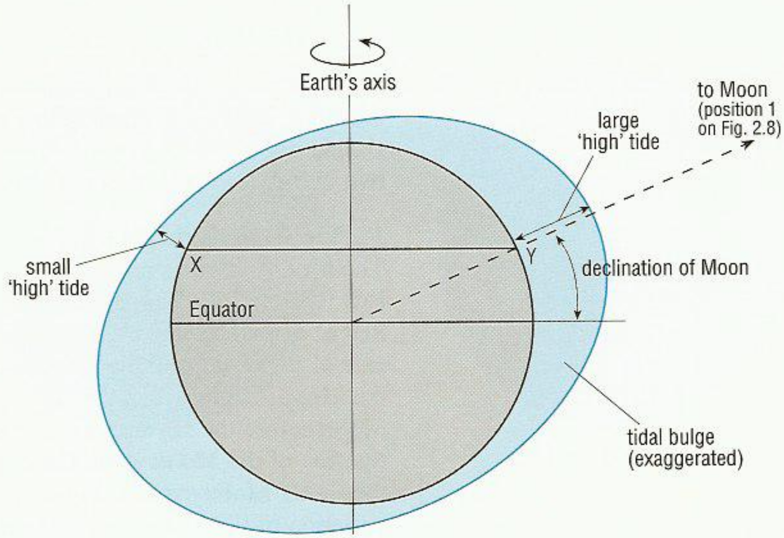

Figure 2.9 The production of unequal tides (tropic tides – see text) at mid-latitudes consequent upon the Moon's declination. An observer at Y will experience a higher high tide than an observer at X; 12 hours and 25 minutes later, their observations will be reversed.

QUESTION 2.3 Assuming that Figure 2.9 shows maximum declination, what will be the extent of the diurnal tidal inequality due to the Moon as seen by a coastal observer at about 28° 30′ south latitude: (a) roughly seven days and (b) roughly 14 days after the situation shown?

Your answer to Question 2.3 emphasizes the cyclical nature of this diurnal tidal inequality. At maximum declination, the Moon is approximately above one of the Tropics (latitude 23.4° N or S), the diurnal inequality is greatest all over the world, and the tides are known as **tropic tides** (Moon at positions 1 and 3 on Figure 2.8); whereas at minimum (zero) declination (when the Moon is above the Equator), there is no diurnal inequality anywhere in the world and the tides are called **equatorial tides** (Moon at positions 2 and 4 on Figure 2.8).

2 The Moon's elliptical orbit

The orbit of the Moon around the Earth–Moon centre of mass is not circular but elliptical, and the Earth is not at the centre of the ellipse, but at one of the foci (Figure 2.10). The consequent variation in distance from Earth to Moon results in corresponding variations in the tide-producing forces. When the Moon is closest to Earth, it is said to be in **perigee**, and the Moon's tide-producing force is increased by up to 20% above the average value. When the Moon is furthest from Earth, it is said to be in **apogee**, and the tide-producing force is reduced to about 20% below the average value. The difference in the Earth–Moon distance between apogee and perigee is about 13%, and tidal ranges are greater when the Moon is at perigee.

2.2 TIDE-PRODUCING FORCES – THE EARTH–SUN SYSTEM

Like the Moon, the Sun also produces tractive forces and two equilibrium tidal bulges. Although enormously greater in mass than the Moon, the Sun is some 360 times further from the Earth, so the magnitude of its tide-producing force is about 0.46 that of the Moon. As we saw in Section 2.1, tide-producing forces vary directly with the mass of the attracting body, but are inversely proportional to the cube of its distance from Earth. The two solar equilibrium tides produced by the Sun sweep westwards around the globe as the Earth spins towards the east. The solar tide thus has a semi-diurnal period of twelve hours.

Just as the relative heights of the two semi-diurnal lunar tides are influenced by the Moon's declination, so there are diurnal inequalities in the solar-induced components of the tides because of the Sun's declination.

The Sun's declination varies over the seasonal yearly cycle, and ranges up to 23.4° either side of the equatorial plane. This angle of 23.4° is the angle between the plane of the Earth's Equator and the plane of the ecliptic (Section 2.1.1) and is therefore also the tilt of the Earth's axis (Figure 2.11).

As in the case of the Moon's orbit round the Earth, the orbit of the Earth around the Sun is elliptical. When the distance between Earth and Sun is at a minimum the Earth is said to be at **perihelion**; when it is at a maximum, the Earth is said to be at **aphelion**. However, the difference in Earth–Sun distance between perihelion and aphelion is only about 4%, compared with an approximate 13% difference in Earth–Moon distance between lunar perigee and apogee. Characteristics of the Earth's orbit round the Sun change cyclically over periods of tens of thousands of years, and these will of course affect the tides, but not on time-scales which concern us for the purposes of this Volume.

QUESTION 2.4 According to Figure 2.11, at what time(s) of the year will the solar-induced component of the tide be at its strongest?

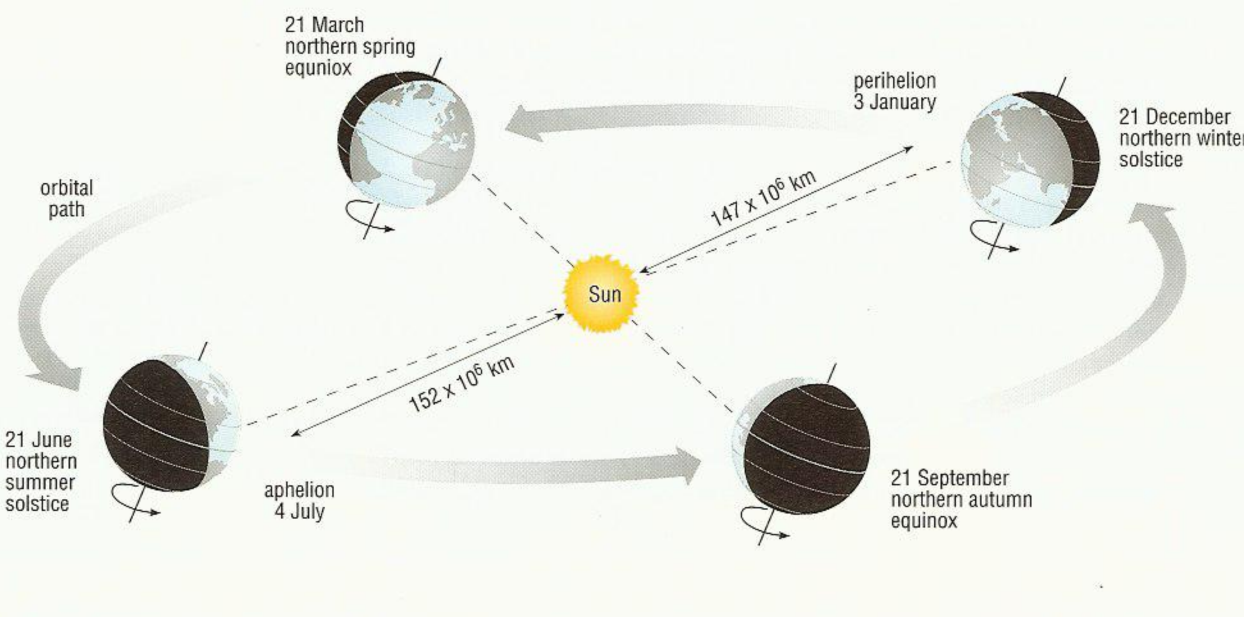

Figure 2.11 The Earth's elliptical orbit round the Sun (not to scale), illustrating four monthly positions corresponding to the seasonal cycle, at summer and winter solstices, and at spring and autumn equinoxes. The plane of the Earth's Equator makes an angle of 23.4° with the plane of the ecliptic (plane of Earth's orbit), so the *tilt* of the Earth's axis is 23.4°, and that is why the Tropics of Cancer and Capricorn are at latitudes 23.4° N and S respectively. The Earth is closest to the Sun in January and furthest away in July.

2.2.1 INTERACTION OF SOLAR AND LUNAR TIDES

In order to understand the interaction between solar and lunar tides, it is helpful to consider the simplest case, where the declinations of the Sun and Moon are both zero. Figure 2.12 (overleaf) shows these conditions, looking down on the Earth from above the North Pole. In Figure 2.12(a) and (c), the tide-generating forces of the Sun and Moon are acting in the same directions, and the solar and lunar equilibrium tides coincide, i.e. they are in phase, so that they reinforce each other. The tidal range produced is larger than the average, i.e. the high tide is higher and the low tide is lower. Such tides are known as **spring tides**. When spring tides occur, the Sun and Moon are said to be either in *conjunction* (at new Moon – Figure 2.12(a)) or in *opposition* (at full Moon – Figure 2.12(c)). There is a collective term for both situations: the Moon is said to be in **syzygy** (pronounced 'sizzijee').

In Figure 2.12(b) and (d), the Sun and Moon act at right angles to each other, the solar and lunar tides are out of phase, and do not reinforce each other. The tidal range is correspondingly smaller than average. These tides are known as **neap tides**, and the Moon is said to be in **quadrature** when neap tides occur. Inshore fishermen sometimes refer to spring and neap tides by the descriptive names of 'long' and 'short' tides respectively.

The complete cycle of events in Figure 2.12 takes 29.5 days and the reason why this cycle is different from the Earth–Moon rotation period of 27.3 days (Figure 2.2) can be seen by reference to Figure 2.13(a) (which is analogous to Figure 2.7). It is simply that in the 27.3 days taken by the Moon to make a complete orbit of the Earth, the Earth–Moon system has also been orbiting the Sun. For the Moon to return to the same position relative to *both* Earth and Sun, it must move further round in its orbit, and that takes an extra 2.2 days or so.

Figure 2.13(b) is a summary diagram of the combined motions of Earth and Moon about the Sun. It shows how both the Moon and the centre of the Earth trace out undulating paths as they themselves rotate about their common centre (the centre of mass of the Earth–Moon system, Figure 2.2). The diagram also illustrates the 29.5-day spring–neap cycle of Figure 2.12, a period sometimes called the *synodic month* but more commonly known as the **lunar month** (i.e. the period between successive new Moons). The 27.3-day period of rotation of the Moon about the Earth–Moon centre of mass is known as the *sidereal month*.

QUESTION 2.5

(a) What is the time interval between two successive neap tides?

(b) What is the state of the tide 22 days after the Moon is in syzygy?

(c) How soon after the new Moon might a tide of 'average' range be expected?

(d) Figure 2.12 illustrates the simplest case of zero declination for both Sun and Moon. Bearing this in mind, what astronomical phenomena would be observed on the Earth's Equator if the Sun, Moon and Earth were in the positions shown in Figure 2.12(a) and (c) respectively?

It is crucial to realize from Figure 2.12 that spring and neap tides must each occur at about the same time *all over the world,* because the Earth rotates *within* the tidal bulges (cf. Figure 2.6), which themselves move only in response to orbital motions of Moon and Earth. For the same reason, the tropic and equatorial tides (Section 2.1.1) must also occur at the same times.

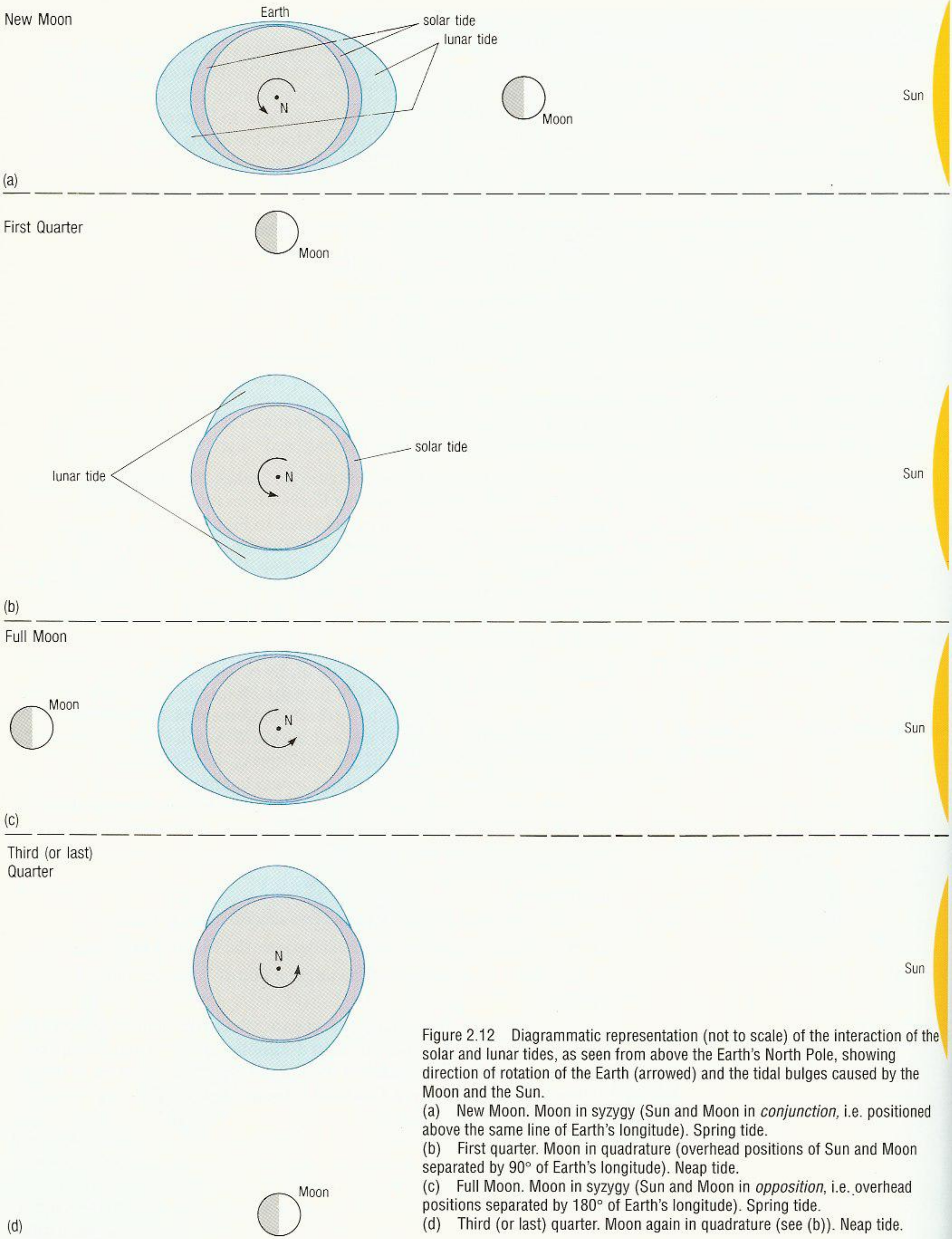

Figure 2.12 Diagrammatic representation (not to scale) of the interaction of the solar and lunar tides, as seen from above the Earth's North Pole, showing direction of rotation of the Earth (arrowed) and the tidal bulges caused by the Moon and the Sun.
(a) New Moon. Moon in syzygy (Sun and Moon in *conjunction,* i.e. positioned above the same line of Earth's longitude). Spring tide.
(b) First quarter. Moon in quadrature (overhead positions of Sun and Moon separated by 90° of Earth's longitude). Neap tide.
(c) Full Moon. Moon in syzygy (Sun and Moon in *opposition*, i.e. overhead positions separated by 180° of Earth's longitude). Spring tide.
(d) Third (or last) quarter. Moon again in quadrature (see (b)). Neap tide.

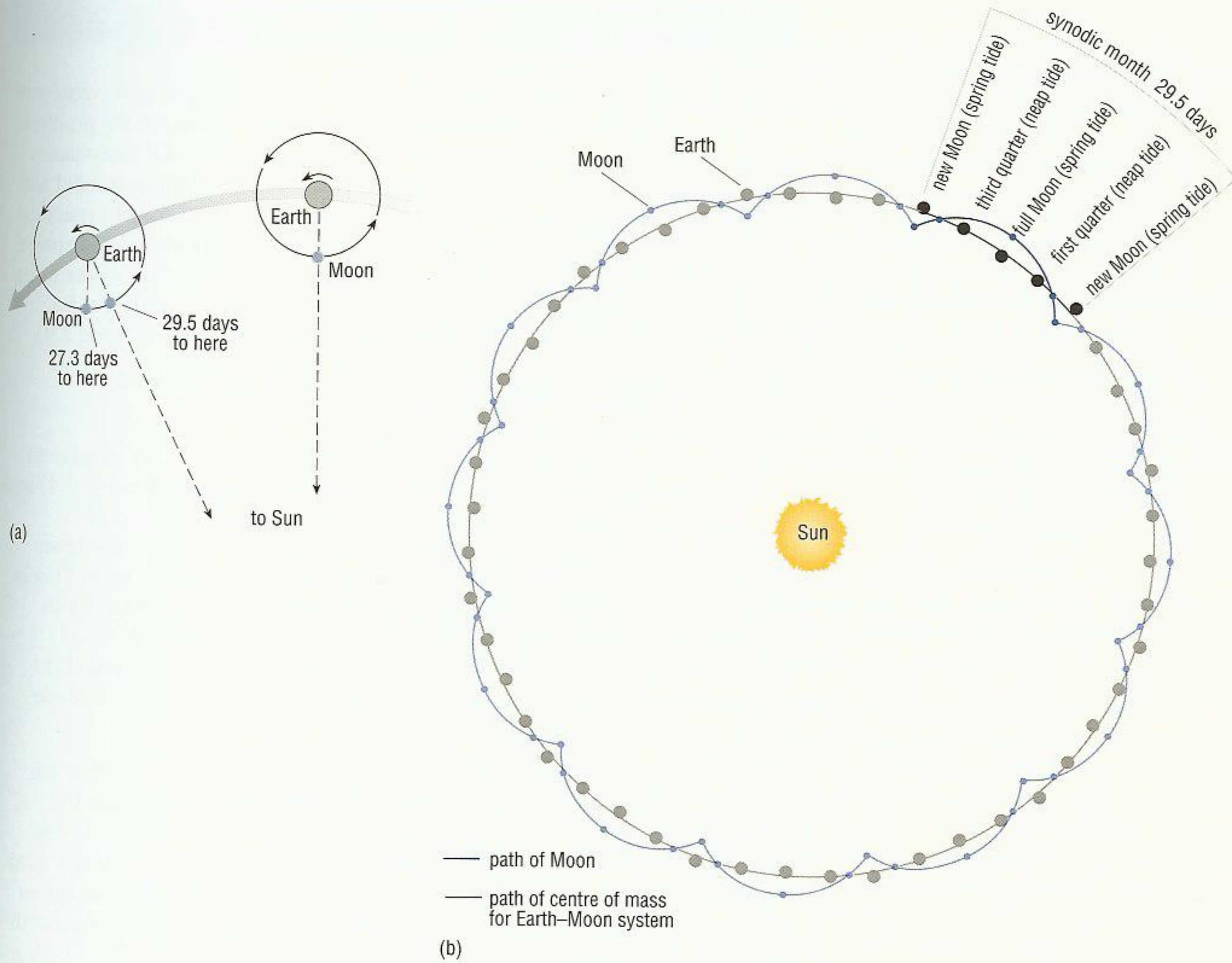

Figure 2.13 (a) Diagram (not to scale), illustrating how the Moon (shown here in conjunction) must travel further round its orbit to return to the same position relative to both Earth and Sun, because the Earth has also moved in its orbit relative to the Sun. For simplicity both orbits are portrayed as circular.
(b) Summary diagram (not to scale) of combined motions of Earth and Moon about the Sun and about the centre of mass of the Earth–Moon system. For simplicity, orbits are assumed to be circular. *Note:* synodic month = lunar month (see p. 63).

The regular changes in the declinations of the Sun and Moon, and their cyclical variations in position with respect to the Earth, produce very many harmonic constituents, each of which contributes to the tide at any particular time and place. One interesting situation is the 'highest astronomical tide', i.e. that which would create the greatest possible tide-producing force, with the Earth at perihelion, the Moon in perigee, the Sun and Moon in conjunction and both Sun and Moon at zero declination. Such a rare combination would produce tidal ranges greater than normal, all over the world. For example, at Newlyn, Cornwall, the normal tidal range is about 3.5 m, the mean spring tidal range about 5 m, and the highest astronomical tidal range about 6 m. However, there is no immediate need to sell any seaside property which you may own – the next such event is not due until about AD 6580.