

### Momentum Equations

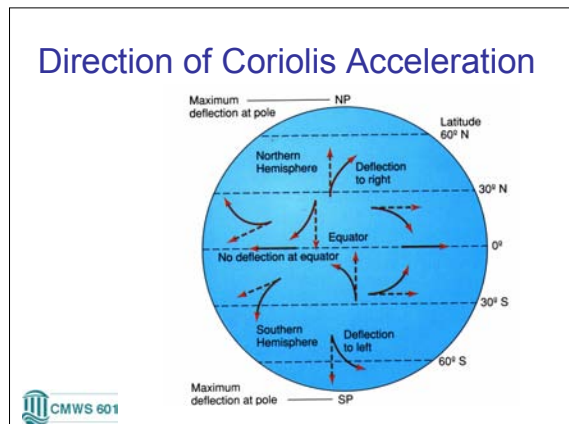
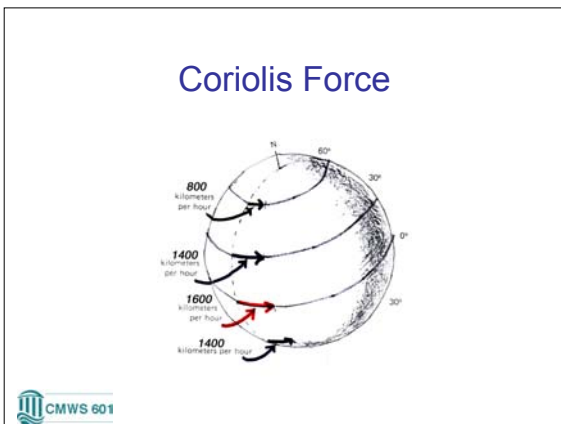
- $F = m a$
- $Acc = F/m$
- $Acc = 1/\rho (F_1 + F_2 + F_3 + F_4 + F_5)$
- $Acc = \text{Coriolis} + \text{Pressure Gradient} + \text{Wind Stress} + \text{Gravity} + \text{Friction}$

### Pressure Gradient Force - accelerates air/water from High to Low

- Air Pressure - mb
- Ocean - db

$$PGF = \frac{1}{\rho} \frac{dp}{dx}$$

- Pressure Gradients arise in the water by differences in horizontal densities
  - Water will flow from high to low pressure
  - Northerly current: to the North
- 



## Coriolis Force = $f \mathbf{V}$

### 1) Latitude Dependent

- Latitude included in the Coriolis Parameter ( $f$ ) :
- $f = 2 \Omega \sin \theta$
- $\Omega = 2\pi / 24 \text{ hrs} = 7.3 \times 10^{-5} \text{ s}^{-1}$
- $\theta = \text{latitude}$

### 2) Velocity Dependent



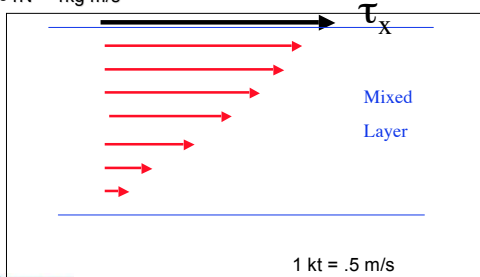
## When is Coriolis important?

- Rossby # =  $v/(f \cdot L) < 1$
- $v$  = velocity
- $f$  = Coriolis parameter
- $L$  = length of motion



## Wind Stress

- At surface  $\tau = c W^2$   $c = 2 \times 10^{-3} \text{ kg/m}^3$
- $1\text{N} = 1\text{kg m/s}^2$



## Wind Stress Force

- The acceleration of water due to a wind stress =  $\frac{1}{\rho} \frac{d\tau_x}{dz}$
- Force acts in the direction of the wind
- Northerly wind: from the North



## Other Forces

- Gravity
  - only acts in the  $-z$  direction
  - $= 9.8 \text{ m/s}^2$
- Friction
  - Smallest force, we will ignore



## Momentum Equations

- $\text{acc} = \text{Coriolis} + \text{Pressure Gradient} + \text{Wind Stress} + \text{Gravity} + \text{Friction}$
- In the x-dir:

$$\frac{du}{dt} = fv + \frac{-1}{\rho} \frac{dP}{dx} + \frac{1}{\rho} \frac{d\tau_x}{dz}$$

If these forces are acting on a water parcel, in which direction are they attempting to accelerate the water?



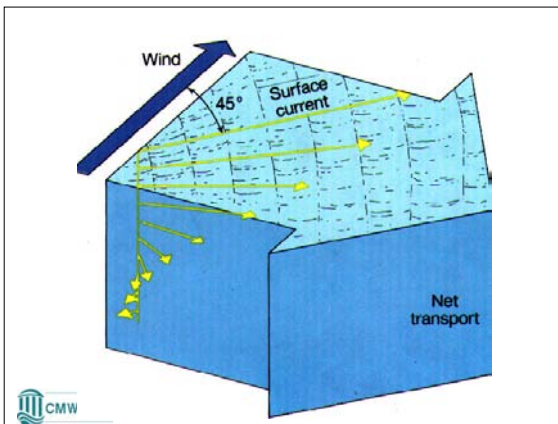
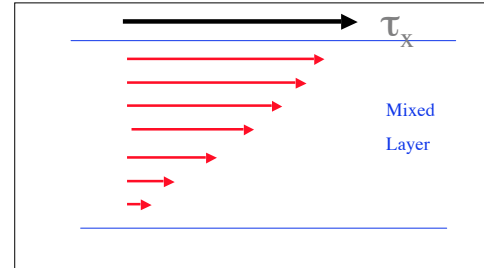
## Ekman Balance

Coriolis = Wind Stress

$$fv = \frac{1}{\rho} \frac{d\tau_x}{dz}$$



## Wind Stress

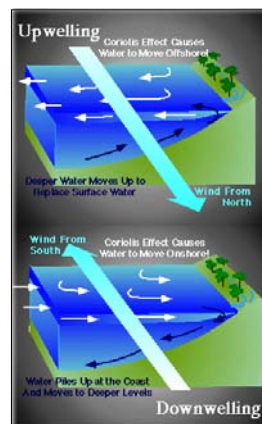
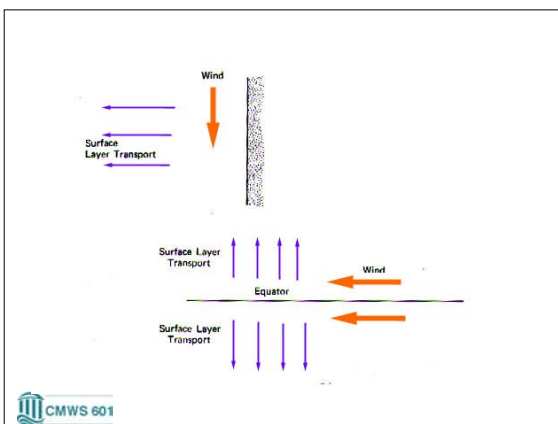


For a wind speed  $W$  (m/s)

Ekman Depth

Surface Current

$$D_e \text{ (m)} = \frac{4.3 W}{(\sin \theta)^{1/2}} \quad V \text{ (m/s)} = \frac{.0127 W}{(\sin \theta)^{1/2}}$$



- Almost all coastal upwelling and downwelling results from Ekman Transport

## Summary: Ekman Balance

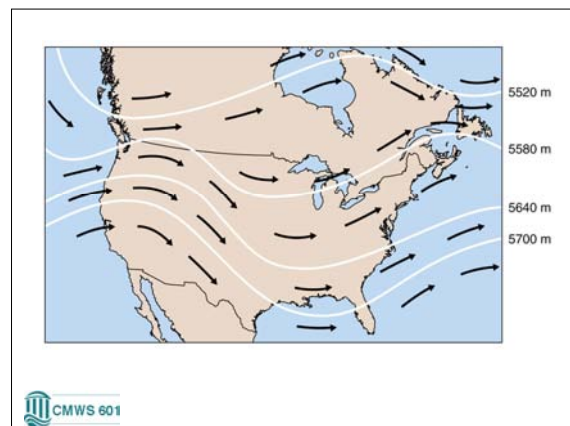
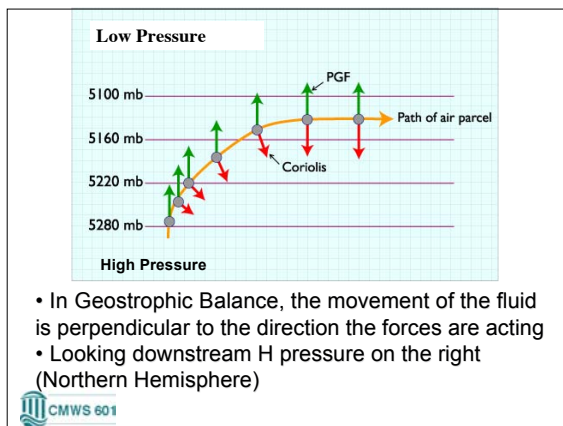
- Is a balance where wind and Coriolis are equal & opposite
- The winds sets a layer of the ocean in motion, Coriolis affects the motion so that in the N. Hemi. the net transport of water is to the right of the wind
- An Ekman spiral is created in the Ekman layer
- Surface divergences created by Ekman transport are the cause of almost all upwelling including coastal upwellings



## Geostrophic Balance

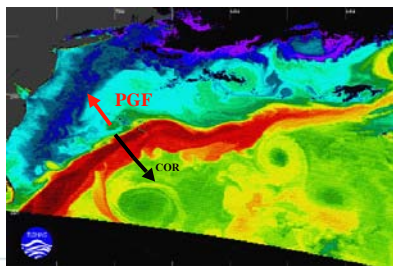
- $PGF = \text{Coriolis}$
- $\frac{1}{\rho} \frac{dP}{dx} = fv$

[Geostrophic Balance Animation](#)

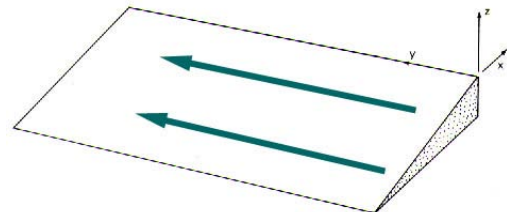


The flow of the current is perpendicular to both of the forces

What creates the PGF?

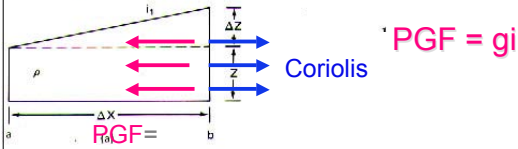


## Along the Hill



How can we get a horizontal PGF in a 1-layer fluid?

1 layer model

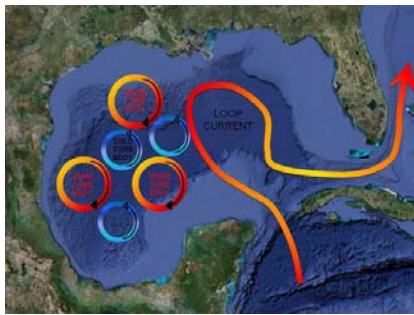


Which direction is the current flowing?



## Geostrophic Balance

- Pressure Gradient = Coriolis
- $f v = \frac{1}{\rho} \frac{dp}{dx}$
- Or for a 1-layer fluid:
- $f v = g i$

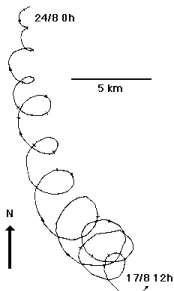
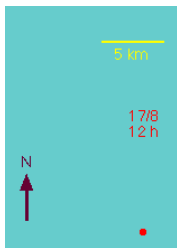


## Summary: Geostrophic Balance

- $PGF = \text{Coriolis}$
- The two forces act perpendicular to the direction of flow
- Where there is flow there is a sea surface slope.
- N. H. : Cyclonic: Low Pressure, CCW



## Inertial Motion



## Inertial Balance

- Acceleration = Coriolis
- $\frac{du}{dt} = f v$
- radius =  $V/f$
- Period (T) =  $2\pi/f$

